MATHEMATICS

Paper 9709/11
Pure Mathematics 1

General Comments

Some very good responses were seen but the paper proved challenging for a number of candidates. For this paper, the knowledge and use of basic algebraic and trigonometric methods from IGCSE or O Level is expected, as stated in the syllabus.

Key messages

The question paper rubric states that 'no marks will be given for unsupported answers from a calculator.' This means that clear working must be shown to justify solutions, and that using calculators to solve equations, writing down only the solution, is not sufficient for certain marks to be awarded. For quadratic equations, for example, it is necessary to show factorisation or use of the quadratic formula (showing values substituted into it) or completing the square. When factorising, candidates should ensure that the factors always expand to give the coefficients of the quadratic equation.

When a question asks for exact answers, this means an answer without rounded decimals (**Question 1(b), Question 5 (a) and (b)**).

It is important that the correct terminology is used when answering transformation questions. A stretch (not 'squeeze' or 'compression', for instance) should always be given with the correct scale factor and with the direction clearly indicated. A translation (not 'move') should be given with a column vector (**Question 8(a)**).

Comments on specific questions

Question 1

(a) The majority of candidates were able to complete the square, obtaining correct values for p and q, though some numerical or sign errors were seen.

(b) Although the question stated 'hence' which implied that candidates should use their result from **part (a)**, most candidates opted to use the quadratic formula instead.

Question 2

Many good attempts at this question were seen, with correct use of the formulae to form equations.

Candidates who chose to use the formula $\frac{n}{2(a+l)}$ for the sum could make no further progress unless they substituted an expression involving a and d for l. Several candidates made arithmetic errors or misinterpreted l as 1, hence reaching incorrect values for a and d. Even with incorrect values, a method mark could be awarded for using the formula for the sum of 50 terms.

Question 3

(a) This question proved challenging for a number of candidates who had difficulty in identifying the correct term to find a, often providing a full expansion which was not necessary. Others made errors in their use of indices, for example writing $(2x^2)^3$ as $2x^6$ or $8x^5$, or gave the requested coefficients as 80 or 24 instead of $80k^4$ or $24k^2$. Some candidates attempted to multiply out five brackets which led to multiple errors.

(b) This part required candidates to form a quartic equation in k which was a hidden quadratic. Successful candidates either factorised the quartic itself or substituted another variable for k^2. Several errors were seen after this step, with some candidates writing $k = \frac{3}{2}$, and others finding only the positive square root or including $k^2 = -\frac{9}{5}$.

Question 4

(a) The best proofs started by combining the two fractions on the left-hand side using a common denominator, with correct notation throughout. Many candidates made a good start but often made sign errors in the numerator or when using $\cos^2\theta + \sin^2\theta = 1$ in the denominator. Others could not factorise the numerator to obtain $\tan^2\theta$ correctly. Some candidates started with the right-hand side instead, though a fully correct solution was harder to achieve as it required more complex factorisation.

(b) Few correct solutions were seen. The majority of candidates used the result of **part (a)** but divided both sides by $\tan^2\theta$ to obtain an equation with no solutions. Those who correctly multiplied out and rearranged the equation sometimes included 0 which was outside the required interval.

Question 5

(a) Most candidates found a correct expression for the area of the sector, though the area of the triangle proved more difficult. It was necessary to subtract the triangle area from the sector area then simplify the expressions. The requirement for the exact area of BCD meant that decimal answers could not be awarded full marks.

(b) Only a very small minority of candidates spotted that the given length of BD in this part was a clue that angle θ was no longer $\frac{\pi}{6}$ but $\frac{\pi}{3}$. For those who continued to use $\frac{\pi}{6}$, a maximum of two marks were available: for using the arc length formula and for using Pythagoras' theorem to obtain the length of AD (and hence CD).

Question 6

(a) Many good solutions were seen, but some candidates had difficulty in factorising out x^2 (or y^2 if they had swapped the variables first). A common error was giving the final result as $\pm\sqrt{\frac{-4x-4}{x-1}}$ instead of just the positive answer. A few candidates chose to divide first and followed the alternative method in the mark scheme.

(b) Most responses showed how to find the given expression correctly, but it was rare for candidates to find both the upper and lower limits of the range. Candidates are reminded to use their calculator with carefully chosen values to help them answer this type of question.

(c) Some strong responses to this question were seen. A correct explanation would mention that part of the range of f (i.e. some of the possible output values) lies outside the domain (or input values) of f.

Question 7

(a) Many candidates correctly equated the expressions for the line and the curve then squared both sides. A very common error was to square the individual terms $\frac{1}{2}x$ and 1. This led to incorrect coordinates for A and B, though a method mark was available for solving a 3-term quadratic.

(b) A correct strategy involved integrating both functions, or the difference between them, or finding the area of a trapezium (or equivalent composite shape) for the area under the line. Errors in integrating $(3x-2)^{\frac{1}{2}}$ to $\frac{2}{9}\ (3x-2)^{\frac{3}{2}}$ were common. It was essential to show a full substitution of limits since answers obtained by calculator use could not be awarded full marks. Arithmetic or sign errors were often seen in the working. A few candidates attempted to square the two functions before integrating.

Question 8

(a) Almost all candidates made errors in describing the sequence of transformations, the most common being to describe one of the transformations in the *x*-direction incorrectly or incompletely. For the horizontal stretch, for example, it was common to give the scale factor as $\frac{1}{2}$ instead of 2 or to state that it was vertical. For the translation, the direction or axis was often incorrect. Candidates should understand that, if the translation is carried out after the horizontal stretch, its vector needs to be $\begin{pmatrix} 60^\circ \\ 0 \end{pmatrix}$ and not $\begin{pmatrix} 30^\circ \\ 0 \end{pmatrix}$.

(b) While many correct responses were seen, this question proved particularly challenging for some candidates who could not rearrange to reach $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$. Others carried out operations in the wrong order to isolate *x* from $\frac{1}{2}x - 30$ or subtracted 30 instead of adding it. A number of candidates supplied only one solution or 30° as their second solution.

Question 9

(a) Most candidates attempted to form an equation of the circle by completing the square on *x* and *y*, then stated the centre and the radius. Sign errors occurred in some cases, either in completing the square or in reading off the coordinates. Numerical errors were common, for example an incorrect value for r^2 in the circle equation. Candidates who sketched a diagram could deduce the lowest point efficiently while others opted to use lengthy algebraic methods. The lowest point was commonly incorrectly shown as (–9, 1) or (–3, 7) in solutions. Some number of candidates appeared to overlook the requirement to find the lowest point.

(b) Some candidates substituted $kx - 5$ into their circle equation from **part (a)** while others used the equation given. It was necessary to show all the steps in collecting terms into a 3-term quadratic equation then apply the condition $b^2 - 4ac > 0$. An alternative valid method involved rearranging their circle equation to find an expression for *y* then equating this to $kx - 5$, but weaker responses did not rearrange first and so could make little progress. Slips in expanding brackets and collecting terms were very common, meaning that only a minority of candidates arrived at a fully correct solution for the inequality.

Question 10

(a) Correct responses substituted $x = -1$ into the 2^{nd} derivative without sign errors and concluded that the stationary point was a minimum.

(b) Most candidates found this question to be particularly challenging, so only a small number of fully correct solutions were seen. Common errors included omitting the constant of integration when finding $\frac{dy}{dx}$ and/or *y*, or substituting $\frac{dy}{dx} = \frac{9}{2}$ (instead of 0). Some candidates did not realise the need to integrate twice. Other errors seen were in attempting to increase powers, for example confusing integration and differentiation, or in simplifying coefficients.

(c) Many candidates set their expression for $\frac{dy}{dx}$ to 0 but were either unable to solve their equation or did not justify the statement given in the question. Some incorrectly equated the 2^{nd} derivative to 0.

(d) Some candidates realised the need to substitute $x = 1$ into their expression for $\frac{dy}{dx}$ but were unable to proceed further because they did not apply the chain rule or those who did use the chain rule often applied it incorrectly.

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Paper 9709/12
Pure Mathematics 1

Key messages

It is very important that candidates understand and become familiar with the general instructions on the front cover of the question paper and the terminology used in specific questions. These key areas include:

- 'Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.' If an exact answer is either required by the question or can be obtained, then a decimal approximation will not be accepted.

- 'You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.' This means that clear working must be shown to justify solutions. For quadratic equations, for example, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for full marks to be awarded. If factorisation is used then candidates should ensure factors always produce the coefficients of the quadratic when expanded. It is also insufficient to simply quote the quadratic formula: candidates need to show values substituted into it. Similarly for integration questions, limits must be clearly substituted.

- The phrase 'Given instead that' means that the information given under **part (a)** of a question and any subsequent answers found for **part (a)**, should not be used or doesn't apply in **part (b)**.

General comments

The paper was generally well received by candidates and a good number of very good responses were seen. The first four questions proved a very accessible start for most candidates. Candidates seemed to have sufficient time to finish the paper. Presentation of work was mostly good, although some of the answers appeared to be written in pencil which is then superimposed with ink, making the writing illegible. Centres are strongly advised that candidates should be told not to do this.

Comments on specific questions

Question 1

This question was a good start to the paper for many candidates who were able to correctly identify and find the required terms from the two expansions. A number of candidates considered the terms rather than the coefficients and became confused with what to do with the resulting x's. it is important that candidates understand the correct meaning of the word coefficient.

Question 2

This question proved to be the most straightforward one on the paper with many fully correct solutions seen. A few weaker responses made an attempt at an arithmetic progression question instead of geometric. A few more obtained a value of r as $\frac{10}{8}$ rather than $\frac{8}{10}$ and then continued to find the sum to infinity, even though it would not exist in this case.

Question 3

This question was also done well, with the vast majority of candidates realising that integration was required, although a few did attempt the question as if it was a straight line rather than a curve. The integration was very often done correctly although a good number did forget to divide the first term by 4 and a few forgot to include the constant of integration. Since the question asked for the equation of the curve it was important that '$y =$' was included.

Question 4

Similarly to **Question 2**, this question was also generally very well done, although a few candidates attempted it as if it was a geometric progression rather than an arithmetic one. Candidates would benefit from practicing a mix of questions where both types of progressions were considered.

Question 5

Both parts of this question proved to be challenging for many candidates. In **part (a)**, most candidates realised the need to equate the curve and the line but many then attempted to use the discriminant rather than substituting the two values given. A few candidates substituted one value into one side of the equation and the other value into the other side or treated 0 as the x value and $\frac{3}{4}$ as the y value. Others did not realise that with two unknown quantities, two equations would be required. In **part (b)**, weaker responses often used the information from **part (a)** or the answers obtained, ignoring the phrase 'Given instead'. Those who understood 'Given instead' were often able to form a quadratic equation in x, use the discriminant equal to 0 and obtain a quadratic in k. The final answers for k were then quite often found using a calculator function and so full marks were not awarded. See '**Key messages**'.

Question 6

This multi-stage question was well done by many candidates. Most realised the need to equate the line and the curve but many then struggled to solve the resulting quadratic equation in $\sqrt{x}$ or as in **Question 5(b)**, did not show the necessary working and used a calculator function instead. Some thought that the answers they obtained were for x rather than $\sqrt{x}$. Many candidates also realised that integration was required for the area under the curve and roughly equal numbers integrated the line or used the area of a trapezium. The integration was usually done correctly but limits need to be clearly seen substituted into the integrand. The final answer was exact so should not have been rounded as per the rubric on the front cover of the paper, but also an exact answer was specifically asked for in this question.

Question 7

Some weaker responses used 2 cm as the length of the line AB rather than the arc, but the vast majority could use the arc length formula to correctly find angle BOA as $\frac{1}{5}$. A number of candidates left this as the answer to **part (a)** or thought that $\frac{1}{6}\pi + \frac{1}{5} = \frac{11\pi}{30}$. In **part (b)** many candidates used the formula for the area of a sector correctly but found the area of the triangle OPB more challenging. The instruction in the question asked for the final answer to be given to 3 significant figures, but this was missed by a quite a number of candidates.

Question 8

Part (a) produced a mixed response with many candidates obtaining two equations in a and b, solving them correctly and subsequently finding the centre of the circle. Weaker responses occasionally worked with only one equation or assumed that AB was a diameter and found the midpoint. In **part (b)** the vast majority attempted to find the gradient of the line joining A to the centre of the circle and then use the negative reciprocal. Some candidates attempted differentiation but with mixed results and some did not give integers in their final answers.

Question 9

Many candidates performed very well on this question, in particular, **part (a)**. Many correct differentials were seen in **part (a)** although these were sometimes simplified incorrectly. In **part (b)** occasionally the second differential was set to 0, but the vast majority knew that it should be the first. Some candidates found solving the subsequent equation challenging and some forgot to find both coordinates. Again, the majority found that the second differential was $\frac{9}{8}$ but a few then concluded that it was a maximum point.

Question 10

Part (a) proved to be one of the most challenging questions on the paper with many candidates omitting it. Many candidates were more confident with the standard inverse function requirements of **part (b)**, although some, having struggled with **part (a)**, missed out the whole of the **Question 10**. In **part (c)** those who found $\mathrm{f}^{-1}(3)$ and then $\mathrm{g}(1)$ were more successful than those who found $\mathrm{gf}^{-1}(x)$ first. Weaker responses sometimes formed an equation = 3 or multiplied the two functions together. A reasonable number of candidates omitted **part (d)** and a significant number of others thought that potentially square rooting a negative number was the issue, rather than the function not being one to one. **Part (e)** was missed out by almost 20 per cent of candidates and those who did attempt it often stopped after the 'show that' part. Others tried to find the area under the curve rather than under the tangent or attempted to differentiate the quotient rather than using the 'show that' part to help.

Question 11

A good number of fully correct solutions were seen for **part (a)**, although sometimes insufficient working was shown for solving the quadratic equation or not all four possible solutions were given. Weaker responses demonstrated a struggle with the original quartic form of the equation and candidates would benefit from more practice on this type of question. **Part (b)** proved to be the most challenging question on the paper with many candidates missing it out and very few good attempts seen. Many candidates ignored the instruction to use the quadratic formula or simply used one value of *k* greater than 5.

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Paper 9709/13
Pure Mathematics 1

Key messages

Candidates are reminded of the importance of following the rubric on the question paper: 'You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.' This was particularly relevant to **Questions 3(b), 5(a)** and **7(b)**. Calculators with the capacity to integrate or differentiate should not be used to provide answers without necessary working, e.g. **Question 8(b)**.

In some questions, candidates need first to consider all possible solutions, then reject any solutions which are not compatible with a given diagram or do not match the conditions stated in the question, e.g. **Question 7(b)** and **11(a).**

General comments

Many candidates appeared to be well prepared for this paper. Most responses showed that advice given in previous reports had been used effectively.

Comments on specific questions

Question 1

This proved to be a good start to the paper for most candidates. Use of the general formula for each term quickly led to the appropriate term and the subsequent algebra was usually completed successfully. Candidates should be reminded of the importance of reading the question carefully. Weaker responses omitted the negative square root when the question had asked for 'values'.

Question 2

Of those candidates who attempted this question, many were able to identify the values of p, q and r. Little working was expected but it was clear they understood that p was connected to amplitude and r to a translation. The correct value of q was seen less frequently, with 2 often given rather than $\frac{1}{2}$.

Question 3

(a) Most candidates realised that they needed to show sufficient working at each stage to justify the given result. Candidates successfully used either form of the formula for the sum to n terms of an arithmetic progression.

(b) A majority of candidates used the result from **part (a)** and the nth term formula with 139 to eliminate d and find n successfully. Nearly all of the correct values of n led to a correct value of d. Almost all answers demonstrated familiarity with the formulae and their use.

Question 4

(a) This part was generally done well. Candidates who transformed the quadratic equation by completion of the square tended to make fewer mistakes. Most knew to add 3 to their expression to effect the translation in the y direction. Many correct responses were seen but weaker answers subtracted 1 from x (rather than adding 1 to x) for the translation in the x direction.

(b) In contrast to **part (a)**, fewer completely correct descriptions were seen in this part. While the question asked for a single transformation, many answers referred to a stretch and a translation. Of those answers where the single transformation was correctly identified as a stretch, a minority of the candidates provided a correct description using a scale factor and direction.

Question 5

(a) This was generally solved correctly by forming a quadratic equation in $\sqrt{y}$. Answers from calculator equation solvers were often seen and these did not receive credit for solving the quadratic equation. The most successful solutions used factorisation, but some fully correct answers were seen from the quadratic formula or completing the square. There were also some answers where the terms in $\sqrt{y}$ were isolated then squared to give a correct quadratic equation in y.

(b) Most correct solutions to **part (a)** led to the four correct angles in this part. Most candidates selected answers from the correct quadrants and rounded decimal answers correctly.

Question 6

(a) The majority of candidates had few problems completing the square to find the given form and many correct solutions were seen.

(b) The candidates who completed this part successfully noted the condition $x < 3$. Answers were usually presented in an acceptable form.

(c) Most candidates correctly changed the subject of their answer to **part (a)**. Those who realised the range of the function must be constrained by $\mathrm{f}^{-1}(x) < 3$ were able to select the negative square root to complete their answer.

(d) Many fully correct answers were seen, obtained either from using $f(2x+4)$ on the answer to **part (a)** or the original equation.

Question 7

(a) Finding the equation of a straight line from the given information proved straightforward, so many correct answers were seen in this part.

(b) Most correct solutions were obtained by solving simultaneously the straight line equation from **part (a)** and the equation of the circle. The algebra was usually completed successfully. Not all candidates realised from the diagram given in the question that only one solution was possible, hence they needed to discard the extra solution. Most responses met the requirement to present the answer in exact form.

Other methods were often used successfully, such as identifying similar triangles or calculating triangle side lengths using the gradient of *BC*. Some candidates produced very elegant solutions.

Question 8

(a) Most candidates differentiated the curve equation, generally with correct results as the fractional powers caused few problems. Although some candidates confused the tangent and normal, many others were able to find the gradient and equation of the tangent correctly.

(b) In this part, most candidates integrated the curve equation successfully. Many found the area under the line and subtracted the area under the curve to reach the correct final answer. Weaker responses did not subtract but left the area under the curve as their answer.

Question 9

(a) Candidates used a variety of methods to find *AC* and angle *A*. The formula for the arc length was well known and almost always used correctly with angle *A* in radians or degrees. Most intermediate answers seen in the working were of sufficient accuracy to reach the correct final answer.

(b) The sector area formula was generally used accurately, again with angle *A* in radians or degrees. There were multiple different methods of finding the area of triangle *ABC* including Heron's formula. Most candidates were aware of the need to complete intermediate calculations to one more degree of accuracy than was required in the final answer.

Question 10

(a) As in **Question 8(b)**, candidates understood the need to integrate and knew how to integrate this type of function. The best answers showed clearly that the value of the integral was zero on substituting the upper limit and also showed how the final result was obtained.

(b) Most candidates were familiar with the requirement to differentiate and proceeded correctly. Those who correctly applied the rates of change of the coordinates at *A* often reached the required cubic equation. Those who chose to expand the cubic term rarely made further progress. In solutions where the cubic expression was made the subject of the equation, it was common for candidates to obtain the correct values of x and y. Weaker responses showed sign or algebraic slips so could not reach the correct values.

Question 11

(a) While the most straightforward approach was to find and equate the gradient of the curve to m, the gradient of the line, some candidates did not realise that the only valid solution was for c and m to be positive. Those who were more successful in finding the correct relationship between c and m solved the equation of the curve with the equation of the line, then used the discriminant. They then used this relationship to find the coordinates of *P*.

Even though the question clearly stated that *P* was a single point, some candidates gave multiple answers. Candidates who produced simple sketches of the graphs using the information given were able to see more clearly what was needed.

(b) This question proved challenging to candidates, many of whom had not reached a correct answer or any answer in **part (a)**. Only a few candidates were able to factorise a quadratic expression with algebraic coefficients to find the coordinates of *Q*.

MATHEMATICS

Paper 9709/21
Pure Mathematics 2

Key messages

Candidates are reminded of the importance of reading a question carefully, both before attempting a solution and after obtaining a solution. This will help them to ensure that all the demands of the question are met, and that answers are in the required form and to the required level of accuracy.

General comments

Most candidates appeared to have sufficient time to complete the paper and space in which to answer the questions. The better prepared candidates were able to perform well, demonstrating their knowledge of the syllabus, using appropriate methods. Many candidates did not appear to be well prepared for some areas of the syllabus and often did not attempt all the questions.

Comments on specific questions

Question 1

(a) Very few correct responses were seen. It was expected that candidates would take logarithms and state that $\ln y = (2x - a)\ln 4$. Some candidates omitted the brackets incorrectly, stating $\ln y = 2x - a\ln 4$. Since this was a 'show that' question, it was essential to identify the gradient of the straight line as $2\ln 4$ before reaching the given answer, $\ln 16$.

(b) Again, very few correct responses were seen. Some candidates who had made poor use of brackets in **part (a)** were occasionally able to make progress. It was expected that candidates equate $-a\ln 4$ to -20.8 and obtain an integer answer. Of those candidates who were able to identify a correct method, many had not read the question carefully which stated that *a* was an integer.

Question 2

(a) Many candidates were able to produce fully correct answers, with most being careful to use θ rather than *x*. Most candidates were able to express $\tan\theta$ and $\cot\theta$ in terms of $\sin\theta$ and $\cos\theta$. A main source of error was using $\sec\theta = \frac{1}{\sin\theta}$ rather than the correct $\frac{1}{\cos\theta}$. The identity for $\cos 2\theta$ was usually applied correctly.

(b) Many correct solutions were seen although some candidates omitted to give a second solution in the given range. Candidates are reminded that final answers in degrees should be given correct to one decimal place.

Question 3

(a) Many candidates attempted to solve the equation $\sin x = \sin 2x$ making use of the correct double angle formula to obtain $a = \frac{\pi}{3}$. Some candidates were able to 'spot' correctly that $a = \frac{\pi}{3}$.

(b) Many candidates used a correct method but errors in the coefficient of $\cos 2x$ were common. Some attempted integration rather than differentiation and so few fully correct solutions were seen.

Question 4

Most candidates who attempted this question realised that implicit differentiation was needed and often attempted the product rule. Some candidates did not differentiate 48 on the right-hand side. Those candidates who did not make use of implicit differentiation were usually able to attempt the equation of a normal using their gradient. A number of good solutions did not give the final answer with integer coefficients. Candidates are reminded of the importance of ensuring that their final answer is in the required form.

Question 5

(a) While many candidates appeared to be aware of the basic shapes of the graphs, it was common to see a modulus graph without a 'V' shape (with no vertex) and the logarithmic curve bending back towards the *x*-axis. Few candidates indicated that the points of intersection of the two graphs represented the only two roots of the given equation.

(b) Candidates should generally be guided by the mark allocation of a question. One mark indicates that there is very little calculation to be done. Few candidates realised that they had to equate $3\ln x$ to $2x-5$, the branch of the modulus graph which gave the larger root.

(c) Most candidates recognised that they needed to form a function such as $\mathrm{f}(x)=x-2.5-1.5\ln x$ or equivalent and then make substitutions of $x=4.5$ and $x=5$ in order to demonstrate a sign change. It was necessary to draw a conclusion after obtaining the two values.

(d) Many candidates were able to identify and use the correct iterative formula to a sufficient number of iterations. Some candidates truncated their final answer to 4.87 whilst others did not give their final answer to the required level of accuracy. It was also evident that some candidates did not know how to approach the question.

Question 6

(a) Most candidates attempted the quotient rule, however many of them used brackets incorrectly or omitted brackets, and this often led to errors in later work. Many candidates did not check their solutions, obtaining $\mathrm{e}^x\left(9\mathrm{e}^{2x}-18\mathrm{e}^x+16\right)=0$ due to a previous sign error. This does not factorise to the given answer.

(b) Many candidates made use of the given result in **part (a)**. Candidates were required to show that there was only one solution to the given equation, so they were expected to state that $\mathrm{e}^x\neq 0$ and $3\mathrm{e}^x+2\neq 0$. Very few candidates did this and as a result their solutions were not fully correct. While many candidates realised that $3\mathrm{e}^x-8=0$, many did not give an exact solution to this equation and often omitted to find the *y*-coordinate. Candidates are reminded to read the question carefully so that they can answer it fully and give their response in the required form.

Question 7

(a) Many candidates attempted algebraic long division and were able to make a good start. Fully correct quotients were frequently seen, together with sufficient work to show that the remainder was 4. Those candidates who made use of synthetic division were usually equally successful.

(b) Very few correct responses were seen. Of the candidates who did attempt this part of the question, most recognised that they needed to make use of the quotient from **part (a)** but often omitted $\dfrac{4}{x+2}$. Some candidates were able to carry out some correct integration but very few of them made further progress.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2

Key messages

Candidates are reminded of the importance of reading a question carefully both before attempting a solution and after obtaining a solution to ensure that all the demands of the question are met and that answers are in the required form and to the required level of accuracy.

General comments

Most candidates appeared to have sufficient time to complete the paper and sufficient answer space in which to answer the questions. Better prepared candidates were able to demonstrate their knowledge of the syllabus, using appropriate methods. Some candidates did not make attempts at all questions and did not demonstrate their understanding on certain areas of the syllabus.

Comments on specific questions

Question 1

Most candidates used the quotient rule successfully to obtain an expression for $\frac{dy}{dx}$. Candidates should be aware that expressions such as $\ln x 2x$ and $\ln x(2x)$ are better written unambiguously as $2x \ln x$.

There were some errors in the manipulation following the substitution of $x = e$ into a correct $\frac{dy}{dx}$. Some candidates did not realise that $\ln e = 1$. Candidates should take care when dealing with expressions such as $\frac{e - 2e\ln e}{e^4}$ and evaluate the numerator first rather than incorrectly attempting to cancel terms in the denominator. Some basic algebra skills were limited in some of the work seen, with some candidates omitting to simplify a final answer of $-\frac{e}{e^4}$. Most candidates gave their answer in exact form as required.

Question 2

(a) Many candidates sketched a correct graph of $y = |2x - 9|$, | but not all realised that the gradient of the graph of $y = 5x - 3$ had to be greater than that of the right-hand branch of $y = |2x - 9|$.

(b) Most candidates knew how to solve a modulus equation, with the linear method generally preferred over the squaring method. However, very few candidates used their graph to select their final answer and most candidates gave an invalid answer in addition to the correct one.

Question 3

It was essential that implicit differentiation was attempted as this was the only possible method. With use of the product rule for $e^{2x} \cos 2y$ and chain rule differentiation of e^{2x} and $\cos 2y$, many responses included sign errors and coefficient errors. Other common errors included not recognising that the product rule was required, omission of a term in $\frac{dy}{dx}$, not differentiating 1 to 0 and also incorrect simplification. However, some completely correct solutions were seen and most candidates attempted to give their answer in an exact form as required.

Question 4

(a) It was expected from the use of ln12 in the question that candidates realised they were required to work with ln1, ln2, ln3 and ln4 and not with decimal equivalents. As the answer ln12 was a given answer, it was essential that full and complete detail showing the manipulation of the logarithms in the final steps of their solution was seen. Most candidates appeared to know the trapezium rule, but the calculation of *h* was sometimes incorrect. Use of decimal equivalents was not acceptable, with several candidates then stating that the decimal answer was the same as ln12.

(b) Many candidates drew an acceptable graph of $y = \ln x$. However, to illustrate the correct reasoning, the graph usually needed to be larger and well-curved so candidates could show clearly the chords forming the top of the three trapeziums.

Question 5

(a) Most candidates used the factor theorem successfully and many went on to factorise p(x) correctly. However, some candidates thought that the quadratic factor obtained after either algebraic long division or synthetic division had to be factorised. It was essential that candidates recognised that this quadratic expression could not be factorised.

(b) Candidates who had factorised correctly in **part (a)** usually went on to use their factors to obtain $e^{3y} = 4$ and from there obtain a correct answer. Candidates who had incorrectly factorised the quadratic factor in **part (a)** should have been alerted to the fact that an error had been made when they were asked to show that there was only one real root to the given equation. It was essential to use the discriminant (or equivalent) to show that the quadratic factor had no real roots in order to obtain full marks. Most candidates attempted to give their answer in exact form as required.

Question 6

(a) Candidates should be aware that it is essential that sufficient detail is shown in a solution when being asked to show a given result.

Most candidates were able to integrate correctly. Some errors were noted when dealing with the indices after the substitution of the limits. Equating the expression to 120 was sometimes forgotten. A common error was to attempt to take logs too early before isolating e^{2a+1}.

(b) Many completely correct iterations were seen, but some responses did not show full details of their iterations to the required level of accuracy. Some candidates truncated their final answer whilst others did not write down their final iteration.

Question 7

Most candidates attempted the correct process required to find the shaded area. Many were successful in completing the correct integration of at least one function. Candidates who did not use the required double angle identity could not proceed with the evaluation of the trigonometrical part.

Errors were sometimes made in the coefficient of $(2\pi - 2x)^{\frac{3}{2}}$. Most candidates attempted to give their final answer in exact form as required.

Question 8

(a) Many candidates obtained the first three marks available for using correct identities. Some sign errors were seen but candidates could still obtain method marks for using a suitable expression, These candidates were usually able to use correct processes and R and α. The range given in the question suggested an answer for α in degrees was required but some candidates gave their answer in radians. It is essential that candidates read the question demands carefully and ensure that they are giving their answers in the correct format and to the correct level of accuracy.

(b) Very few correct solutions were seen. Better solutions identified that a substitution of 2β for α in their result from **part (a)** was needed, but few were able to obtain an accurate answer at this stage.

Some candidates attempted to make a fresh start, not making the obvious link between the two parts as indicated by use of the word ‘Hence’.

MATHEMATICS

Paper 9709/23
Pure Mathematics 2

Key messages

Candidates are reminded of the importance of reading a question carefully both before attempting a solution and after obtaining a solution to ensure that all the demands of the question are met and that answers are in the required form and to the required level of accuracy.

General comments

Most candidates appeared to have sufficient time to complete the paper and sufficient answer space in which to answer the questions. Better prepared candidates were able to demonstrate their knowledge of the syllabus, using appropriate methods. Some candidates did not make attempts at all questions and did not demonstrate their understanding on certain areas of the syllabus.

Comments on specific questions

Question 1

Most candidates used the quotient rule successfully to obtain an expression for $\frac{dy}{dx}$. Candidates should be aware that expressions such as $\ln x 2x$ and $\ln x(2x)$ are better written unambiguously as $2x\ln x$.

There were some errors in the manipulation following the substitution of $x = e$ into a correct $\frac{dy}{dx}$. Some candidates did not realise that $\ln e = 1$. Candidates should take care when dealing with expressions such as $\frac{e - 2e\ln e}{e^4}$ and evaluate the numerator first rather than incorrectly attempting to cancel terms in the denominator. Some basic algebra skills were limited in some of the work seen, with some candidates omitting to simplify a final answer of $-\frac{e}{e^4}$. Most candidates gave their answer in exact form as required.

Question 2

(a) Many candidates sketched a correct graph of $y = |2x - 9|$, | but not all realised that the gradient of the graph of $y = 5x - 3$ had to be greater than that of the right-hand branch of $y = |2x - 9|$.

(b) Most candidates knew how to solve a modulus equation, with the linear method generally preferred over the squaring method. However, very few candidates used their graph to select their final answer and most candidates gave an invalid answer in addition to the correct one.

Question 3

It was essential that implicit differentiation was attempted as this was the only possible method. With use of the product rule for $e^{2x}\cos 2y$ and chain rule differentiation of e^{2x} and $\cos 2y$, many responses included sign errors and coefficient errors. Other common errors included not recognising that the product rule was required, omission of a term in $\frac{dy}{dx}$, not differentiating 1 to 0 and also incorrect simplification. However, some completely correct solutions were seen and most candidates attempted to give their answer in an exact form as required.

Question 4

(a) It was expected from the use of ln12 in the question that candidates realised they were required to work with ln1, ln2, ln3 and ln4 and not with decimal equivalents. As the answer ln12 was a given answer, it was essential that full and complete detail showing the manipulation of the logarithms in the final steps of their solution was seen. Most candidates appeared to know the trapezium rule, but the calculation of h was sometimes incorrect. Use of decimal equivalents was not acceptable, with several candidates then stating that the decimal answer was the same as ln12.

(b) Many candidates drew an acceptable graph of $y = \ln x$. However, to illustrate the correct reasoning, the graph usually needed to be larger and well-curved so candidates could show clearly the chords forming the top of the three trapeziums.

Question 5

(a) Most candidates used the factor theorem successfully and many went on to factorise p(x) correctly. However, some candidates thought that the quadratic factor obtained after either algebraic long division or synthetic division had to be factorised. It was essential that candidates recognised that this quadratic expression could not be factorised.

(b) Candidates who had factorised correctly in **part (a)** usually went on to use their factors to obtain $e^{3y} = 4$ and from there obtain a correct answer. Candidates who had incorrectly factorised the quadratic factor in **part (a)** should have been alerted to the fact that an error had been made when they were asked to show that there was only one real root to the given equation. It was essential to use the discriminant (or equivalent) to show that the quadratic factor had no real roots in order to obtain full marks. Most candidates attempted to give their answer in exact form as required.

Question 6

(a) Candidates should be aware that it is essential that sufficient detail is shown in a solution when being asked to show a given result.

Most candidates were able to integrate correctly. Some errors were noted when dealing with the indices after the substitution of the limits. Equating the expression to 120 was sometimes forgotten. A common error was to attempt to take logs too early before isolating e^{2a+1}.

(b) Many completely correct iterations were seen, but some responses did not show full details of their iterations to the required level of accuracy. Some candidates truncated their final answer whilst others did not write down their final iteration.

Question 7

Most candidates attempted the correct process required to find the shaded area. Many were successful in completing the correct integration of at least one function. Candidates who did not use the required double angle identity could not proceed with the evaluation of the trigonometrical part.

Errors were sometimes made in the coefficient of $(2\pi - 2x)^{\frac{3}{2}}$. Most candidates attempted to give their final answer in exact form as required.

Question 8

(a) Many candidates obtained the first three marks available for using correct identities. Some sign errors were seen but candidates could still obtain method marks for using a suitable expression, These candidates were usually able to use correct processes and R and α. The range given in the question suggested an answer for α in degrees was required but some candidates gave their answer in radians. It is essential that candidates read the question demands carefully and ensure that they are giving their answers in the correct format and to the correct level of accuracy.

(b) Very few correct solutions were seen. Better solutions identified that a substitution of 2β for α in their result from **part (a)** was needed, but few were able to obtain an accurate answer at this stage.

Some candidates attempted to make a fresh start, not making the obvious link between the two parts as indicated by use of the word ‘Hence’.

MATHEMATICS

Paper 9709/31
Pure Mathematics 3

Key messages

- It is important that candidates write clearly, crossing out rather than erasing incorrect working. They should avoid overwriting pencil solutions with ink as the result is usually illegible.
- Candidates' responses should match the demand of the question, in particular giving answers to the accuracy asked for.
- If a question asks for an exact answer then a decimal approximation will not be accepted.
- Candidates should use mathematical notation correctly, in particular including brackets when appropriate and not using equals signs between lines that are not equal.

General comments

Some candidates offered solutions to all of the questions, presenting logical arguments, but many candidates attempted to answer questions on just a few selected topics.

It was clear from the large number of attempts at long division in **Question 5(a)** that many candidates are not familiar with either the remainder theorem or factor theorem. The majority of candidates did not appear to be familiar with the method for integration by substitution (**Question 6(a)**). Very few candidates had a method for finding the tangent of the angle between the two lines in **Question 8(b)**.

Comments on specific questions

Question 1

Many candidates started their response with the incorrect statement $2\left(3^{2x-1}\right)=6^{2x-1}$. The majority of solutions did include the correct statement $\ln\left(4^{x+1}\right)=(x+1)\ln 4$. Some candidates who obtained a linear equation in *x* could not see how to solve this to obtain the value of *x*. A few candidates were successful in reducing the given equation to $\left(\frac{3}{2}\right)^{2x}=6,$ which is relatively straightforward to solve.

Question 2

(a) There were a few fully correct solutions. The most common errors were in taking out the factor of 2 incorrectly, either not obtaining 2^{-2} or going on to expand $\left(1-x^2\right)^{-2}$. Candidates with a correct unsimplified expansion often made errors in the arithmetic when simplifying the terms.

(b) Many candidates did not attempt this question. Some candidates gave part of the correct answer, but not the full set of values.

Question 3

The majority of candidates expanded tan2*x* and many used the formula to obtain a correct quadratic equation in tan*x*. The most common errors seen were in the algebra when simplifying to obtain the quadratic equation. Some candidates made a false start with equations such as $\frac{1}{2}\tan 2x+\frac{1}{3}\tan x=\frac{1}{5}.$ A few candidates confused cot*x* with cos*x*. Candidates with a correct quadratic in tan*x* often gave the acute answer but overlooked the obtuse answer.

Question 4

Most candidates who showed understanding of differential equations started with correct separation of the variables. They usually integrated to $\ln y$ correctly. Some candidates misinterpreted the integral with respect to x as an inverse tangent. Several candidates obtained a correct solution in terms of logarithms, but the final step to obtain a simplified expression for y was more challenging. Those candidates who attempted to remove logarithms before finding the constant of integration usually obtained a constant of an incorrect form.

Question 5

(a) Many candidates used particularly long and inefficient methods to answer this question. Many attempted to use long division, but often did not reach a constant remainder. A more efficient approach would be to use the Factor Theorem with $\mathrm{p}(x)$ and $\mathrm{p}'(x)$ to give a simple pair of simultaneous equations in a and b.

(b) Very few candidates were aware that, if $(x-2)$ is a factor of both $\mathrm{p}(x)$ and $\mathrm{p}'(x)$, then $(x-2)^2$ is a factor of $\mathrm{p}(x)$, so they needed to use division or an alternative method to factorise the cubic expression. Some candidates had clearly used a calculator to find the roots of the equation and then tried to work back to the factors but did not always include the 3.

Question 6

(a) Very few candidates made any attempt to use $\frac{\mathrm{d}x}{\mathrm{d}\theta}=3\sec^2\theta$. Some of those who did differentiate stated $\frac{\mathrm{d}\theta}{\mathrm{d}x}=3\sec^2\theta$ instead. Some candidates attempted to substitute for x in the integral but could not proceed because they did not use the relationship $1+\tan^2\theta=\sec^2\theta$. Most candidates who substituted correctly for x also dealt correctly with the limits. Some lengthy methods used sines and cosines but were generally unsuccessful in reaching the given answer.

(b) This is a standard integral and those candidates who recognised the need to use the double angle substitution usually completed the task successfully. It is possible to use integration by parts twice, but candidates who attempted this approach did not get as far as the second phase of integration.

Question 7

(a) Many candidates started by multiplying the numerator and denominator by $1-2\mathrm{i}$. Some of them obtained a denominator of 5 but there were several numerical and algebraic slips in simplifying the numerator. A small number of candidates took the slightly simpler approach of multiplying $x+\mathrm{i}y$ by $1+2\mathrm{i}$ and comparing the real and imaginary parts.

(b) Only a minority of candidates was familiar with the form $re^{\mathrm{i}\theta}$ and how this relates to $x+\mathrm{i}y$.

(c) A few candidates followed through their answer to **part (b)** correctly. Those who did not reach an answer to **part (b)** could not attempt this part of the question.

Question 8

(a) Many candidates attempted the implicit differentiation. Even though they did not have the help of a given answer, several of them obtained a correct expression for the derivative.

(b) This was an unusual question, and many candidates did not know where to start. Some candidates realised that if they wanted to find the angle between the two tangents then it would be useful to start by finding their gradients. Some candidates simply substituted $x=0$ and $y=0$ without thinking about the other coordinates of the two points. Of the small number of candidates who obtained two correct gradients, very few had a method for finding the angle between the two lines. Although asking the candidates to find the tangent of the angle had been intended to provide a hint about a suitable method, those candidates who attempted to use the scalar product came closest to a correct result.

Question 9

(a) Several candidates gave correct expressions for the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$. Most errors were due to slips in arithmetic or misinterpretation of the diagram.

(b) Many candidates were aware of the correct form for the equation of the line and they used their answers to **part (a)** correctly. The most common error was in writing the equation without $\mathbf{r} = \ldots$.

(c) This part of the question was more challenging. Some candidates who were at least partially successful understood that they needed to start by finding the coordinates of the foot of the perpendicular from the origin to the line. Several fully correct solutions were seen.

Question 10

(a) Many candidates recognised the need to use the product rule for differentiation. Those who could also use the chain rule correctly usually reached the given answer. The majority of errors occurred in attempts to differentiate $\sqrt{\sin x}$.

(b) There were several fully correct solutions. Most errors were due to not working in radian mode.

(c) The majority of candidates did not understand what was required here. A few stated a correct equation without subscripts, but very few explained the crucial step $\tan\theta = -\tan(\pi - \theta)$.

(d) The majority of candidates who attempted this working in radian mode reached the correct conclusion.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3

General comments

The majority of candidates offered solutions to all the questions. There were a few candidates who could make little progress, but many others demonstrated a very good understanding of the topics covered.

Some questions, such as Trigonometry (**Question 4**) and Vectors (**Question 9**) presented a challenge because they were slightly different to anything the candidates might have seen before. Several candidates made multiple attempts to obtain the given answer in **Question 9**. Two questions, Numerical methods (**Question 5**) and Complex numbers (**Question 10**) required candidates to draw a basic sketch and this proved challenging for many candidates.

Key messages for candidates

- Candidates should be prepared to answer questions on the whole of the syllabus.
- It is important that candidates write clearly, crossing out rather than erasing incorrect working. They should avoid overwriting pencil solutions with ink as the result is usually illegible.
- Candidates' responses should match the demand of the question, in particular giving answers to the accuracy asked for.
- If a question asks for an exact answer then a decimal approximation will not be accepted.
- Candidates should use mathematical notation correctly, in particular including brackets when appropriate and not using equals signs between lines that are not equal.
- If a question involves both calculus and trigonometry then candidates should be working in radians.
- Where more than one mark is available, an answer that is not supported by any working is likely to be insufficient.

Comments on specific questions

Question 1

Many correct responses were seen to this straightforward starter question. Those candidates who followed a correct method almost always obtained the correct answer, with just a few rounding their answer incorrectly. The most common error was an incorrect initial statement such as $\ln\left(e^{2x}+3\right)=\ln e^{2x}+\ln 3$. Some candidates were confused between $e^{2x}+3$ and e^{2x+3}.

Question 2

The majority of candidates used a correct double angle formula to replace $\cos 2\theta$ then obtained a correct quadratic in $\cos\theta$. Some arithmetic slips were seen in obtaining the quadratic equation, but the majority of errors were in solving the resulting trigonometric equation. Many candidates obtained the correct basic angle $45.9°$. Some candidates using an approximation to the root of the quadratic frequently gave an incorrect value for the angle. Several candidates overlooked the fact that with a negative value for $\cos\theta$ they should be looking for solutions in the second and third quadrants.

Question 3

There were many fully correct responses to this question. Those candidates who substituted $x = \frac{1}{2}$ and $x = -2$ into the polynomial had more success than those who attempted to divide by $2x - 1$ and $x + 2$. Long division rarely resulted in a correct solution. A common mistake was to substitute $x = 3$ into the polynomial instead of $x = -2$ (from $5 - 2 = 3$). A few candidates made sign errors and arithmetic slips in solving for *a* and *b*.

Question 4

The differentiation in this question proved to be quite challenging. Most candidates made some attempt to use the product rule. Correct use of the chain rule was much less common, either because candidates simply did not apply it (e.g. $\frac{\mathrm{d}}{\mathrm{d}x}\cos^3 x = \sin^3 x$) or because they had errors in the coefficients. After the differentiation, many candidates very succinctly showed the steps to obtain the correct answer via a horizontal equation with positive powers of sin *x* and cos *x*. For other candidates, there were many algebraic errors in dealing with $\sqrt{\sin x}$ and other errors in obtaining an equation in only one trigonometric function. The question makes it clear that an answer in radians is required, but candidates should also know to work in radians after differentiating a trigonometric function. Although the numbers involved were simple, several accuracy errors were seen in the final answer.

There were two alternative methods used by some candidates, one being by first finding the logarithm of the function ($3\ln\cos x + \frac{1}{2}\ln\sin x$), they obtained a function that was simpler to differentiate. Another method of dealing with the square root was to start by squaring and then differentiating.

Question 5

(a) Many candidates produced incorrect graphs or graphs that omitted the necessary features. The majority of sketches did not have the *y*-axis as an asymptote for $y = \ln x$. On many sketches $y = 3x - x^2$ did not pass through the origin, and on others it was the wrong way up. Candidates who did plot the two graphs correctly could not be awarded both marks if they made no reference to the root.

(b) Many candidates gave fully correct responses to this part. Candidates should be reminded to make their conclusion clear, especially when finding and comparing four values.

(c) The majority of candidates chose an initial value in the interval [2, 2.8] and demonstrated correct use of the iterative process. The calculations in **part (b)** suggest that the root is closer to 2.8 than to 2 but there was little evidence of candidates making use of this. Some candidates did not give values to the required level of accuracy, and a few reached a false conclusion by completing insufficient iterations. A minority of candidates showed enough iterations to justify the root but did not then give their final answer to 2 decimal places as required.

Question 6

(a) Candidates who separated the variables correctly and recognised the two integrals usually reached the solution to this differential equation and gave the answer in a simplified form. There was a significant number of candidates who never expressed e^{y-x} as $\mathrm{e}^{y} \times \mathrm{e}^{-x}$ and so could not proceed correctly. Sign errors were common in integrating both functions. The most common incorrect integral was $\int \mathrm{e}^{-y}\mathrm{d}y = \ln\left(\mathrm{e}^{y}\right)$. A few candidates used the product or quotient rule to differentiate $x\mathrm{e}^{-x}$ instead of using integration by parts. Those candidates who tried to take logarithms before evaluating the constant of integration usually used a constant of an incorrect form and could not reach a correct answer. There were many errors in the final step from $\mathrm{e}^{-y} = x\mathrm{e}^{-x} + \mathrm{e}^{-x}$ to an expression for *y*, the most common being $\ln\left(\mathrm{e}^{-y}\right) = \ln\left(x\mathrm{e}^{-x}\right) + \ln\left(\mathrm{e}^{-x}\right)$.

(b) Most candidates with a correct answer to **part (a)** answered this part correctly.

Question 7

(a) Many candidates gave fully correct solutions to this part of the question. The given answer helped some to identify sign errors in their working, or to realise they needed to differentiate the right-hand side of the equation. Candidates should be reminded that, when working towards a given answer, their working needs to be fully correct. Any error needs to be traced back to its origin.

It was common for candidates to introduce $\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ at the start of their working when differentiating implicitly. The majority omitted it later, but some candidates did not and this led to incorrect results.

A significant minority of candidates was not familiar with implicit differentiation and so could make no progress. A few of these candidates tried to rearrange the equation to separate the variables.

(b) Most candidates understood the need to set the numerator of the given derivative equal to zero. Several errors were seen in forming an equation in x or y, the most common being $3 \times 2^2 = 6$. Those candidates who obtained a correct equation usually obtained the point $(-2,1)$, but as the question makes it clear that more than one point is expected, some of them assumed that the other point would be $(2,-1)$. Others used complex numbers to find the other two cube roots of 1. The most common error to overlook the factor of x in the numerator, or to reject it as not relevant.

Question 8

(a) For many candidates this part of the question was very straightforward. Those candidates who substituted carefully selected values of x often completed the task efficiently. Candidates who set up three simultaneous equations were more prone to errors. A surprising number of candidates who reached the correct equation $10 = 4 + 2B + 6$ did not conclude that $B = 0$. Some candidates seemed suspicious of reaching $B = 0$ and made a second attempt before being convinced.

A significant minority of candidates started incorrectly with a constant numerator over the quadratic denominator. They needed to demonstrate that the coefficient of x was 0 rather than assume it.

(b) Most candidates recognised that integration of $\frac{1}{3x-1}$ led to a natural logarithm. Many of them gave an answer in the correct form, although often with an incorrect coefficient. Of those who had the correct $\frac{3}{x^2+3}$ from **part (a)**, a significant minority thought that the integral would be another logarithmic term. Of those who recognised the form required, most correctly reached $k\tan^{-1}\frac{x}{\sqrt{3}}$ but were less successful in finding the correct coefficient. Candidates with a non-zero B value from **(a)** very rarely made any progress with the second integral.

Several candidates used degrees when evaluating the definite integral. Candidates need to be aware that they should use radians when working with trigonometric functions in calculus problems.

Question 9

(a) Although quite simple, this task was an unfamiliar one and proved challenging to many candidates. It was made more challenging when candidates lacked a clear strategy or made multiple errors in the arithmetic and the algebra. Those candidates who used the component equations to eliminate the parameters λ and μ, obtaining an equation in a and b, often made good progress.

(b) Those candidates who started with the correct scalar product equation usually reached a fully correct answer. The most common errors were in the arithmetic and algebra. A significant minority of candidates did not link the word 'perpendicular' with the direction vectors or with scalar product.

(c) This was a very accessible task for those candidates who attempted it. The answer to **part (b)** gave them the values they needed. Depending on their approach in **part (a)** they often had an equation for λ or μ in terms of a or b ready to use.

Question 10

(a) Candidates had a variety of approaches to **part (a)**, the most common being to substitute $-1+\sqrt{7}\text{i}$ into the equation and solve for *k*. They needed to show working for the expansion of the brackets and not just use a calculator. The most concise way to expand the brackets was to use the binomial expansion, but for some candidates this led to multiple errors in the arithmetic and algebra. Some candidates combined their answers to **parts (a)** and **(b)** by finding the quadratic factor first. Those who did this were often successful in presenting clear, succinct solutions.

(b) Many candidates recognised that $-1-\sqrt{7}\text{i}$ is also a root of the equation. The majority then used this root to form the quadratic factor which they divided into the cubic. In some cases, sign errors in the quadratic factor and an incorrect value of *k* led to incorrect answers. There was some confusion between the terms 'root' and 'factor', with $(2x-1)$ often stated as a root of the equation. A significant minority of candidates saw the word 'root' and attempted to find the square roots of $-1+\sqrt{7}\text{i}$.

(c) Most candidates were able to identify the correct centre and radius of the circle. Some candidates had non-linear scales on their axes or unequal scales on the two axes, so they obtained an ellipse.

(d) Successful candidates usually worked from a carefully annotated diagram and showed clear working. The question makes it clear that an answer in radians is required.

MATHEMATICS

Paper 9709/33
Pure Mathematics 3

Key messages

Candidates need to:

- show all steps in the working when attempting a question with a given answer – **Questions 5(c)**, **6(a)** and **10(a)**
- know what is required when showing points on an Argand diagram – **Question 5(a)**
- understand how to solve vector questions related to straight lines – **Questions 9(b)** and **9(c)**
- realise that working is required in complex number questions – **Question 5(b)**. Candidates are reminded that calculators cannot be used in complex number questions since they need to show working to justify their result, as stated in the rubric.

General comments

The standard of work on this paper was very high, with a considerable number of candidates performing well on most of the questions. Candidates were generally aware of the need to show sufficient working in their solutions, especially in **Questions 10(a)**, **10(b)** and **10(c)**.

Some candidates' work was difficult to read. Candidates should be reminded to set out their work clearly to ensure it is legible.

Comments on specific questions

Question 1

Despite the algebraic symbol *a* adding difficulty to this question, most candidates provided fully correct solutions. The preferred approach was to square the equality, solve the resulting quadratic equation and choose the region between the two roots. Some candidates opted to present just the final answer following their correct quadratic equation. They should realise, in this situation, a response with limited working is only creditworthy if the solution is correct.

Question 2

Many candidates provided correct answers using the correct trigonometric formula with the correct values of sin60 and cos60. However, a common error was to show tan θ expressed as $6-\sqrt{3}$ instead of its reciprocal, either through a simple algebraic error or mistakenly thinking that tanθ was $\dfrac{\cos\theta}{\sin\theta}$. The alternative approach of using $R\cos(\theta \pm \alpha)$ was not often seen.

Question 3

(a) Most candidates produced the correct quadratic equation with no errors. Others who dealt correctly with the coefficient of the $2\log_3(x-1)$ term, and converted 1 to $\log_3 3$, combined these two terms incorrectly as $3+(x-1)^2$ instead of $3(x-1)^2$.

(b) A considerable number of candidates found this question difficult. Some candidates carried through an incorrect quadratic equation from **part (a)**. Others solved for x instead of y, hence had twice the required solution. Many candidates included the root that should have been rejected. Finally, there were candidates who gave an exact solution instead of a decimal one or did not present their answer to 2 decimal places as required.

Question 4

(a) Although there were many correct answers to this part, some candidates appeared to guess values for a and b from the form requested in the question instead of showing detailed supporting working. A few candidates did not realise that they needed to use the product rule for differentiation, only differentiating the exponential term. Others made an error in their coefficient of $\sin 2x$ and finished with 4 instead of -2.

(b) This part of the question proved relatively straightforward for most candidates, who were assisted by being given the $\sin 2x$ form in **part (a)**. A few candidates only found one of the two solutions, while others worked in degrees rather than radians.

Question 5

(a) Most candidates correctly evaluated u^* and $u^* - u$, but some did not show any scale on their Argand diagram when plotting the positions of points A, B and C. It is essential to have a scale, equal on both axes, to indicate the correct positions of these points. The question asked candidates to state the type of quadrilateral formed, and while nearly all knew it had to do with parallel lines, few were able to supply the correct term 'parallelogram'. Other candidates omitted to state the type of quadrilateral altogether.

(b) Many excellent solutions were seen, however, as mentioned earlier, working must be shown. Although the calculations involved were not difficult, it was necessary for candidates to show the steps in the method rather than writing down an answer from the calculator.

(c) Many good answers to this question were seen. Better responses showed sufficient detail, such as relating $\arg\dfrac{u^*}{u}$ to $\arg(u^*) - \arg(u)$, together with $\arg(u) = -\tan^{-1}\dfrac{1}{3}$. Some candidates omitted to show the necessary signs in their working to justify the given answer. With errors or omissions candidates could not reach a fully correct argument.

Question 6

(a) In this question, it was common for candidates to substitute the independent variable x for t. Such notation errors meant that candidates could not construct a complete argument to reach the given answer. The derivative of x could be written down from the List of Formulae MF19, but candidates were required to apply the chain rule to find the derivative of y. Most candidates successfully combined $\dfrac{dy}{dt}$ and $\dfrac{dx}{dt}$ correctly to obtain $\dfrac{dy}{dx}$ and many of them produced fully correct solutions.

(b) Since the answer to **part (a)** was given, there were many correct solutions seen in **part (b)**. However, some candidates confused x and t in the equation of their tangent. Others used the gradient of the normal to the curve rather than the gradient of the tangent.

Question 7

(a) Almost all candidates reached partial fractions in the correct form with correct coefficients. Errors were seen in choosing a constant term as the numerator for the $(2x^2 + 3)$ denominator, or through equating coefficients then solving incorrectly the equations formed.

(b) Whilst many fully correct solutions were seen, many candidates made basic errors. These errors usually arose through difficulties in expressing $(x-2)^{-1}$ as $-\dfrac{1}{2}\left(1-\dfrac{x}{2}\right)^{-1}$ and $(2x^2+3)^{-1}$ as

$\frac{1}{3}\left(1+\frac{2x^2}{3}\right)^{-1}$. It was common for the coefficient, when extracted from the bracket, to appear incorrectly in the numerator rather than the denominator.

Question 8

(a) There were some accurate and clear solutions to what was a challenging differential equation question. Most candidates separated the variables correctly and integrated both sides successfully.

Other candidates made coefficient or sign errors in their $N^{-\frac{1}{2}}$ term and $\sin(0.02\,t)$ term. A common error led to 0.02 in the numerator rather than the denominator. Whilst other errors in integrating $\cos(0.02\,t)$ were relatively rare, some errors were seen in integrating $N^{-\frac{3}{2}}$ to $N^{\frac{1}{2}}$ or even $\ln N$.

(b) Most candidates made a good attempt to evaluate *k,* but some were let down by poor algebra, such as inverting term by term. Others incorrectly used degrees instead of radians.

(c) Again, candidates found the algebra difficult and struggled to convert to an expression for *N* in terms of *t* with their value of *k* substituted. Many very basic errors were seen; hence it was rare to see a correct value for the maximum value of *N*.

Question 9

(a) Many fully correct solutions were seen, but too often inappropriate vectors were used. Some candidates calculated the magnitude of a different pair of vectors from the ones they used to find the scalar product.

(b) Candidates should always draw a clear diagram to represent a geometrical situation. This would have avoided the very common error of setting to zero the scalar product of **OP** and the direction vector of *l*.

(c) In this part, a diagram would have clearly indicated that the reflection vector could easily be found by adding the vector **OP*** found in **part (b)** and the vector **AP*.**

Question 10

(a) Excellent work was seen in this question. The integration by parts was nearly always correct, with the result manipulated into the given answer together with full working.

(b) Errors were rarely seen, although some candidates who compared values of *a* and $\left(\frac{35}{3\ln a-1}\right)^{\frac{1}{3}}$ discussed sign changes instead of the crossover of these two functions between $a = 2.4$ and $a = 2.8$.

(c) The majority of candidates produced fully correct answers. These included all the details of working to 4 decimal places, as well as sufficient iterations to confirm convergence.

MATHEMATICS

Paper 9709/41
Mechanics

Key messages

- Non-exact numerical answers are required correct to 3 significant figures or correct to 1 decimal place for angles, as stated on the front of the question paper. Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- When answering questions involving any system of forces, a well annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium equation or a Newton's Law equation. This was particularly noticeable in **Questions 2**, **3** and **7**.
- In questions such as **Question 6** in this paper, where velocity is given as a function of time, then calculus must be used and it is not possible to apply the equations of constant acceleration.

General comments

The paper was well attempted by many candidates although a wide range of marks was seen.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst providing challenge for the stronger candidates. **Questions 1(a), 1(b)**, **4** and **7(a)** were found to be the most accessible questions whilst **Questions 5(c)**, **6(b)** and **7(b)** proved to be more demanding for candidates.

In questions such as **Question 3** where the sine of an angle is given, it is not necessary to evaluate the angle. In fact, doing so may lead to approximations which could affect the accuracy.

Comments on specific questions

Question 1

(a) This question was well answered by most candidates. The most straightforward method to find the time taken is to use $s = \frac{(u+v)}{2}t$ which gives the value of *t* immediately. A different approach is first to find the acceleration using $v^2 = u^2 + 2as$ and then to find *t* using $v = u + at$.

(b) Most candidates made a successful attempt at this question. Even those who had made an error in **part (a)** could be awarded both marks using their value for *t*. A few candidates drew a trapezium with curved sides.

(c) In this part candidates had to first find the total distance travelled by the car during its motion and then divide this by the total time taken. The simplest way of finding this distance was to find the area of the trapezium drawn in **part (b)** although many candidates chose to add the two given distances to the area of the triangle of height 25 and base 5. A significant number of candidates made errors in finding the distance.

Question 2

This question involves two connected particles moving vertically upward. It was attempted by most candidates, with many correct responses seen. It is necessary to apply Newton's second law, either to each of the two particles and solve simultaneous equations, or to the system as a whole to find the acceleration and then to one of the particles to find the tension. The first approach was more popular but often sign errors were introduced. Candidates who used the second approach usually found the acceleration correctly, though some made an error in finding the tension.

Question 3

In this question the exact value of $\sin\alpha$ is given. Candidates are not expected to evaluate the angle as this leads to a slight loss of accuracy. The question involves a crate which is in limiting equilibrium under the influence of an unknown force *X*. In order to find *X*, it is first necessary to find the normal reaction force in terms of *X* and then to resolve horizontally to find the friction force as a component of *X*. Finally, an equation in *X* can be formed and solved using $F = \mu R$. The most common errors were to omit the term in *X* from the normal reaction force, to use the wrong component in resolving or to introduce a sign error, usually $R = 300g + 0.28X$. Some very good responses this question were seen.

Question 4

Most candidates attempted this question, either by resolving forces in two perpendicular directions or by using the sine and cosine rules. Most candidates resolved horizontally and vertically while some chose to resolve in the directions parallel and perpendicular to the 100 N force. A few errors were seen such as sign errors or confusing sine and cosine. The most straightforward method for finding *F* is to square and add the $F\sin\alpha$ and $F\cos\alpha$ values and then take the square root. To find α the easiest method is to divide the values of $F\sin\alpha$ and $F\cos\alpha$ to obtain the value of $\tan\alpha$. Most candidates used the above methods. Those who correctly found the angles $(\alpha + 20)°$ and 20°, almost always successfully found the values of *F* and *α*. A few candidates attempted to use the sine and cosine rules, and they were rarely successful.

Question 5

(a) Many candidates did not realise that, since they were given the total work done against friction rather than the friction force, they needed to use an energy approach. Those who did identify this usually found the correct given answer, by equating the gain in kinetic energy to the difference between the work done by the driving force and the work done against the resistance force.

(b) Many correct responses were seen in this part. It was possible use an energy method, which few candidates adopted, or to apply Newton's second law which the majority of candidates used successfully. This required them to find the acceleration and then equate the difference between the driving force and the resistance to motion to the mass multiplied by the acceleration. The most common error with this approach was to in the acceleration. The energy method requires the gain in kinetic energy to be equated to the difference between the work done by the driving force and the work done against the resistance force.

(c) Candidates generally found this part challenging, with few attempting it. The first step is to find the power of the car's engine at *P* using Power = *F* × *v* (80 000 W) and then to equate the driving force at this power $\frac{80\,000}{v}$ to the resistance of 1200 N. Many candidates who attempted the question made an error in finding the power, often using the value of 2000 N from **part (a)** as their *F* rather than the correct 3200 N. A wide variety of other incorrect methods was also seen.

Question 6

(a) In this question, it is necessary to integrate to find an expression for displacement since the velocity is given as a function of *t*. Many candidates realised this and correctly found the displacement in terms of *t*. Having found this expression, there are several different approaches to verifying that the particle returns to *O* at *t* = 2. For example, this can be done by using limits 0 and 2 in the expression, or by including a constant of integration, *C*, showing *C* = 0 then substituting *t* = 2. The majority of candidates used one of these two approaches, but those who included a constant often did not evaluate it. A rather different approach is to solve the equation $k\left(t^3 - \frac{1}{2}t^4\right)$ to show that the solutions are 0 and 2 only.

(b) This part was found to be rather challenging, with some candidates omitting it altogether and relatively few fully correct solutions seen. In order to find the total distance, the first step is to find the time at which the velocity is zero. Candidates who attempted this part often found this time correctly. The next step is to differentiate to find an expression for acceleration, then substitute the value of t just found into this expression and equate it to the given acceleration of $-13.5\ \mathrm{m s^{-2}}$, to find the value of k. Finally, it is necessary to evaluate the expression for distance found in **part (a)** between limits 0 and 1.5, using the value of k previously found. The total distance is twice this value since the displacement from $t = 0\,\mathrm{s}$ to $1.5\,\mathrm{s}$ is positive and from $t = 1.5\,\mathrm{s}$ to $2\,\mathrm{s}$ is negative. An alternative is to use limits 0 and 1.5, and then 1.5 and 2, adding together the moduli of the two results. Candidates who found the time at which the velocity is zero often then found the correct value of k. However relatively few of these managed to find the final answer correctly. Common errors here were to use incorrect limits, e.g. 0 and 2, which leads to an answer of zero.

Question 7

(a) This was a straightforward application of the principle of conservation of momentum. Most of the candidates who attempted this part wrote down the required equation and solved it to find the speed of particle B.

(b) Candidates found this part very challenging and many of them did not attempt it. It concerns the motion of the two particles in **part (a)** after their first collision. The two particles are on an inclined plane. Particle B hits a reflecting barrier and then returns up the plane to collide again with particle A. Candidates are first required to show that the velocity of particle B immediately after hitting the barrier is $0.5\ \mathrm{m s^{-1}}$. To do this, it is necessary to find the speed of B at the instant at which it hits the barrier ($5\ \mathrm{m s^{-1}}$) then reduce this by 90 per cent. There are several methods of finding the speed: using $s = \frac{(u+v)}{2}t$ to find the speed directly; finding the acceleration then using $v = u + at$ or $v^2 = u^2 + 2as$; using an energy method, although this was rarely seen.

It is necessary to find the velocities of the two particles immediately before their second collision. This involves finding the acceleration ($g\sin 30°$) then using $v = u + at$ to find the velocities of the particles, noting that for A, $u = 2.5$, $a = 5$ and $t = 0.44$ but, for B, $u = -0.5$, $a = -5$ and $t = 0.04$. Those very few candidates who correctly found the two velocities were then almost always able to use the principle of conservation of momentum once more to find the speed of the combined particle. In the better responses, the acceleration was found correctly. The velocities of the two particles before their second collision were sometimes incorrect as a result of sign errors or an incorrect value of t. Weaker responses often did not realise that they needed to find the acceleration, although some successfully showed the required velocity of B after hitting the barrier.

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium equation or a Newton's Law equation. Such a diagram would have been particularly useful in **Questions 2**, **3** and **5**.
- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

General comments

The questions were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1** and **6(b)** were found to be the most straightforward questions whilst **Questions 4, 6** and **7(b)** proved to be the most challenging for candidates.

In **Question 6(a)**, the angle θ was given exactly as $\sin^{-1} 0.12$. There is no need to evaluate the angle in problems such as this as any approximation of the angle can lead to a loss of accuracy in the answer.

The rubric on the front of the question paper instructs candidates to take $g = 10\ \text{ms}^{-2}$ and it was noted that almost all candidates did this. In fact, in some cases, such as in **Question 3(a)** in this paper, it was impossible to achieve a correct given answer unless this value was used.

Comments on specific questions

Question 1

(a) This question was answered well by the majority of candidates. It is necessary to use the principle of conservation of momentum for the collision between particle *A* and particle *B*. The majority of candidates wrote down the correct form of equation, but several candidates incorrectly used $\frac{1}{4} \times 8.5$ for the speed of *A* after the collision. On a few occasions, conservation of Kinetic Energy was used instead of conservation of momentum.

(b) Most candidates successfully calculated the kinetic energy of one of the particles. Those who only considered the loss in energy of particle *A*, or who approximated prematurely, could not reach a correct answer for the loss in kinetic energy for the system. A small number of candidates did not know the definition of kinetic energy.

Question 2

Many candidates made good attempts at this question. For a fully correct solution, they needed to state correctly the direction in which the resultant acted. It was common for candidates to resolve horizontally and horizontally, although the clarity and layout of their working was often very poor. Most knew how to find the magnitude of the force and a correct angle. To make the direction of the force clear, a simple vector diagram with an angle indicated would have been sufficient. The direction could also be stated as a rotation through a certain angle from a fixed direction, for example a clockwise rotation through 64.8° from the 14 N force.

Question 3

(a) This was a straightforward pulley question to which many candidates produced good answers. Most candidates correctly wrote down the two Newton's Second Law equations. Those who reached a negative value for the acceleration did not always state the magnitude as requested. The majority of candidates showed the tension correctly, not assuming the given answer. One error seen was approximating the value of acceleration when finding the tension. This resulted in a tension that was either slightly greater or slightly less than 16. Another error was giving the value of acceleration to 2 significant figures.

(b) This part of the question proved to be more challenging for candidates. It was necessary to find the speed of the particles as *A* reached the ground using the constant acceleration equations. Many candidates thought that the particles fell with acceleration $a = \frac{10}{3}$ for 1.5 m instead of 2.1 m. Some found the time taken to reach ground level but then, rather than use this to find the speed, they evaluated the distance travelled which was already known. After moving 2.1 m, particle *B* moves under gravity until reaching its greatest height. However, many candidates continued to use $a = \frac{10}{3}$ in this stage of the motion. A minority of candidates, having done all the correct work, gave the height gained by particle *B*, rather than the greatest height of *B* above the plane.

Question 4

This was found to be the most challenging question on the paper for most candidates.

(a) A significant number of candidates appeared confused by the 4 second time lag, sometimes assuming the wrong particle was 4 seconds ahead. Others used t as the time from when particle *A* passed through *O* rather than particle *B*. Another common error seen was the use of the same variable t for both particles.

(b) The equation to be solved when setting $s_A = s_B$ depended on the approach taken in **part (a)**. Candidates who used the correct definition of *t* usually went on to find the correct solution. Those who had used *t* for both particles often had a more complicated equation to solve, leading to errors when expanding the term $20(t \pm 4) - (t \pm 4)^2$.

(c) Very few candidates produced fully correct graphs. The quadratic graph representing particle *B* was often shown as two straight lines. The straight line representing particle *A* was often shown starting at the origin rather than from a point on the positive *y*-axis. Some candidates sketched two correct graphs but needed to annotate their axes with $s = 32$, $t = 4$ and $t = 8$.

Question 5

This question was fairly well attempted by many candidates, almost all of whom resolved parallel and perpendicular to the inclined plane. This gave two-term expressions for both the friction, F, and the normal reaction, R. Some errors with signs were seen and also interchanging of the sine and cosine components. Some candidates wrongly thought that the normal reaction was either $R = 120\cos 24$ or $R = 120$. The final answer was often given as $\mu = 0.05$ rather than the correct answer to three significant figures as requested in the rubric on the front page of the paper.

Question 6

(a) This question also proved challenging to many candidates. A significant number of candidates thought incorrectly that they could use the equations for constant acceleration, which is not the case. With a constant power, the speed of the car is changing, hence the driving force produced by the engine of the car varies and the car's acceleration is not constant. The work-energy principle must be used here. It is necessary to find the work done by the engine, the increase in kinetic energy and the increase in potential energy. Combining these correctly in the work-energy equation will give the required work done against the resistive forces. Some errors were seen in trying to find a driving force from the given power and using one, or both, of the given speeds to find the work done by the engine. Some candidates used an incorrect change in height in calculating the potential energy. Several candidates incorrectly gave the change in kinetic energy as $\frac{1}{2}\times 900\times(16-11)^2$ instead of $\frac{1}{2}\times 900\times\left(16^2-11^2\right)$.

(b) Many candidates produced good solutions, realising that the given power could be used to find an expression for the driving force. Since the car was travelling at constant speed, this driving force is exactly balanced by the resistance. Errors in the algebraic manipulation were common, for instance in forming a quadratic equation in v with incorrect signs in the coefficients. Another common error was candidates miscopying 32 000 in their own working as 3200 at a later stage. A small proportion of candidates gave both 20 and –400 as final solutions.

Question 7

(a) This question required candidates to consider the acceleration at $t = 10$ from both expressions for velocity and compare the values. Many candidates differentiated the expressions for velocity correctly, and reached the correct answers from their accelerations, but did not go on to complete a convincing argument. Another error was integrating instead of differentiating the expressions for v.

(b) This question required candidates to realise that the velocity is negative from $t = 12$ to $t = 20$. Most candidates correctly found the distance travelled from $t = 0$ to $t = 10$, by evaluating the area of a triangle, by using constant acceleration formulae or by integration. However, most candidates integrated the second stage from $t = 10$ to $t = 20$ without considering the sign of the velocity.

MATHEMATICS

Paper 9709/43
Mechanics

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the front of the question paper, rather than to two significant figures or three decimal places, e.g. **Question 6(a)**.
- When answering questions involving forces in equilibrium or Newton's Second Law or an energy approach, a complete force diagram can be helpful to ensure that all relevant terms are included in the equations formed, e.g. **Question 5(c)**, **Question 6(a)** and **Question 6(b)**.
- When calculating distance travelled using integration, candidates are advised to consider whether the time interval needs to be split when applying limits, e.g. **Question 7(b)**.

General comments

This paper was generally well attempted by candidates with many scripts of a very high standard. **Question 1**, **2(a)** and **5(b)** were found to be the most straightforward to candidates. **Question 4(b), 6(b)** and **7(b)** were the most challenging questions.

Comments on specific questions

Question 1

(a) This question proved to be straightforward with most candidates equating the momentum before and after the collision in order to find the speed of *P*. A few candidates mistakenly attempted to use conservation of energy instead of conservation of momentum. Occasionally a velocity of $-2\ \text{ms}^{-1}$ rather than a speed of $2\ \text{ms}^{-1}$ was given as the final answer.

(b) Most candidates understood that particles *Q* and *R* moved together after the collision. The most common errors were to confuse the velocity or mass of particle *Q* with those of particle *P*.

Question 2

(a) The most usual method of solution was to apply $v^2 = u^2 - 2gs$ from the time of projection until the maximum height was reached. Errors were unusual in this part of the question.

(b) This part of the question was found to be more challenging. Candidates knew how to apply the constant acceleration formulae for motion under gravity and found the time taken for various stages of the motion correctly, but frequently misread or misinterpreted the time required. The simplest solution depended on realising that the time required was double the time taken to decelerate from the projection speed to a speed of $10\ \text{ms}^{-1}$. Some candidates found the time for which the speed of *P* was less than $10\ \text{ms}^{-1}$ rather than '*at least* $10\ \text{ms}^{-1}$'.

Question 3

(a) Most candidates interpreted the graph correctly and calculated the speed as the gradient of the line: $\dfrac{150-50}{10-5}\ \text{ms}^{-1}$. A few attempted to calculate different speeds for $t = 5$ and for $t = 10$ rather than the speed **between** $t = 5$ and $t = 10$.

(b) Given that the acceleration was constant between points O and P, many candidates used $s = ut + \frac{1}{2}at^2$ successfully to find the acceleration. Those who involved v in the constant acceleration formula (e.g. $v = u + at$) often incorrectly assumed constant speed when calculating the final speed as $\frac{50}{5} = 10\text{ms}^{-1}$ leading to $a = 2\text{ ms}^{-2}$.

(c) This question was often solved accurately by dividing the total distance by total time. Some candidates omitted to include the 200 m return to O. Others attempted to find the area of the triangle under DE for the return journey, treating this part of the graph as a velocity-time graph. An alternative method seen was to average the average speeds for each section of the graph. In this case it was necessary to use equal time intervals and to use the speed for each interval rather than including a negative velocity for the final five seconds.

Question 4

(a) Most candidates resolved the forces horizontally and vertically and solved the resulting simultaneous equations as expected. Those who used 0.6 m and 0.8 m to find the exact values for $\sin A$, $\cos A$, $\sin B$ and $\cos B$ were able to obtain exact values for the two tensions while those who found approximate values for the angles at A and at B (36.9° and 53.1°) were more likely to have an error due to premature approximation in the tension values. Some candidates oversimplified the situation, either by assuming that the tensions in the strings were equal or by assuming angles of 45° at A and at B.

(b) This was a more challenging problem which was not attempted by a significant number of candidates. The solution depended on realising that, for the greatest value of F, the tension in the string BC was zero and the tension in the string AC had changed. The problem could then be solved using the resolved equations from **part (a)** with 20 N replaced by F N and the tension in BC replaced by zero. A number of candidates resolved using F and the tensions found in **part (a)** to obtain the original force of 20 N instead of the greatest value of F.

Question 5

(a) The question required the use of Newton's Second Law and $P = Fv$, with F representing the driving force. The power could be found by first calculating the driving force and then applying $P = Fv$. An alternative approach was to form an equation in P directly using $\frac{P}{v} - R = ma$ or $P = (ma + R)v$.
Most candidates formed and solved equations accurately. A few candidates used $P = 70 \times 0.3$ without considering the resistance or used $P = 30v$ without considering the acceleration. Another error seen was to use $F = 70g + 30$ rather than $F = 70 \times 0.3 + 30$.

(b) Nearly all candidates calculated the change in kinetic energy correctly. An error seen occasionally was evaluating $\frac{1}{2}m(v-u)^2$ instead of $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$.

(c) The distance d could be found by applying 'KE gain = PE loss – work done against the resistance'. This was sometimes applied with a sign error or with the force 30 N rather than the work done $30d$ J. A common error was to oversimplify the situation, equating the kinetic energy gain to the potential energy loss. A few candidates included the change in potential energy, $70gd\sin 5$, and the work done by gravity, $70g\sin 5 \times d$, without realising that these are the same.

Question 6

(a) The most usual method of solving was to apply Newton's Second Law to each particle so as to obtain simultaneous equations in a, the acceleration of the particles, and T the tension in the string. The equation for particle Q was sometimes seen as $0.2g\sin 30 - T = 0.2a$ instead of $0.2g\sin 30 + T = 0.2a$. A force diagram should indicate that the tension in the string and the component of weight down the plane for Q act in the same direction. Some candidates used a correct method to set up and solve the equations but gave the final answers as $a = 7.2$, $T = 0.438$, following premature approximation, or two significant figures ($T = 0.44$) instead of the required three significant figures.

(b) This part of the question was found to be more challenging. The situation involves forces in equilibrium with a changed tension in the string. The new tension could be found by resolving along the plane for particle P ($T = 0.3g\sin 60$). Some candidates used the tension from **part (a)**, whilst others omitted the tension when resolving along the plane for particle Q. The additional force of 3 N was sometimes missing from the equilibrium equation. There was uncertainty about the direction of the frictional force which occasionally resulted in a negative coefficient of friction. Some candidates chose to solve the problem twice, with the frictional force acting up the plane and then acting down the plane. This was appropriate as long as they selected the correct solution. A commonly seen incorrect solution had the frictional force in the wrong direction and used the tension found in **part (a)**. The equation obtained was $3 - \mu(2\cos 30) = 0.439 + 2\sin 30$, leading to $\mu = 0.901$. A few solutions included '*ma*' although the particles were at rest, or mistakenly included the normal reaction at P as well as at Q when applying $F = \mu R$.

Question 7

(a) The majority of candidates knew that differentiation of v was needed to find the acceleration. Those who made an error when differentiating, usually in the term $\frac{b}{(t+1)^2}$, could not then achieve the given result. Many candidates obtained correct simultaneous equations for b and c using the given velocity and acceleration for $t = 5$. Since this was a 'show that' question with $b = 9$ given in the question, it was expected to see working out for solving these simultaneous equations rather than calculator values stated.

(b) This question involved integration and the use of suitable limits to find the distance travelled. Candidates often found $\int v\,dt$ successfully although some found difficulty with $\int \frac{b}{(t+1)^2}\,dt$. It was common to see limits of 0 and 10 leading to a value for displacement rather than the distance travelled. The question stated 'the velocity of P is zero only at $t = 5$' which was an indication that $\int_0^5$ ... and $\int_5^{10}$ ... were needed. Some candidates assumed mistakenly that the value of the integral was zero when $t = 0$.

MATHEMATICS

Paper 9709/51
Probability & Statistics 1

Key messages

Candidates must be aware of the requirement to show the necessary steps to support their answer, particularly if they are asked to show a given result. Candidates should ensure they communicate clearly when combining different stages in their solution to provide a final answer or giving a reason to support their comment.

Where a diagram is required it should be clear, accurate and labelled appropriately.

Candidates should state only non-exact answers correct to 3 significant figures; exact answers should be stated exactly. To justify a final answer correct to 3 significant figures, values correct to at least 4 significant figures should be used throughout the calculations. There is no requirement for fractions to be converted to decimals.

General comments

Where candidates make more than one attempt at a question, they should identify clearly which attempt they intend to be marked. When the additional page is used the question number should be clearly shown.

Successful candidates often used diagrams, sketches and tables to both inform and support their explanations.

Many good solutions were seen in **Questions 1**, **3** and **6** where the context was clearly understood. **Question 5** was found to be challenging, with some candidates unable to make a start on **parts (b)** and **(c)**.

Candidates appeared to have sufficient time to attempt the questions, although a few candidates did not appear to be well prepared for all areas of the syllabus.

Comments on specific questions

Question 1

Most candidates identified the need to use factorials to find arrangements.

(a) Many candidates correctly calculated the number of arrangements as 5! or ${}^5P_5$ but this was sometimes not evaluated. Some candidates thought that the Es and Ds were distinguishable and so divided by 2! or 3! or both.

(b) This question presented a common scenario and many candidates produced efficient and clear solutions. Others realised that 8! was involved in finding the total number of arrangements but did not divide by 3! and 2! to take into account the 3 Es and 2 Ds. Successful solutions demonstrated the need to put the 3 Es together and treat them as a single item, leaving 6 items (of which 2 are Ds) to be arranged. Some candidates only considered 5 items so could not reach the correct answer.

Question 2

(a) Many solutions did not take into account that one of Mr Lan or Mrs Lan must be included and chose to exclude them both. An incorrect assumption seen was that only 3 places on the committee remained to be filled by the other 12 members. Some candidates used a rather lengthy method of considering the men and women on the committee separately. Here, too, it was common to see only 3 places being considered. Very few responses were seen with ${}^{13}C_4$ but attempts with ${}^{12}C_4$ were sometimes seen. Of these, many did not multiply by 2 to signify that either Mrs or Mr Lan should be included.

(b) The best solutions provided correct answers and also listed clearly the possible scenarios, indicating how they calculated the number of combinations. Sometimes the scenarios were insufficiently explicit, e.g. selecting 5 women but not specifically including Mrs Lan. In some instances the scenario of Mrs Lan and four women was omitted. Some candidates included Mrs Lan when selecting the women and calculated ${}^{8}C_r$ and ${}^{6}C_{4-r}$. Others added rather than multiplied the number of ways of selecting the men and the women in each of the possible scenarios.

Question 3

(a) Many candidates drew a bar chart instead of a histogram while others attempted a cumulative frequency diagram. Better responses stated the frequency densities before drawing the histogram. Candidates would be well advised to adopt this approach to assist with plotting accuracy. A suitable scale must be selected to permit all the data to be represented, in this case 1 cm representing 10 units. Most candidates used the correct class intervals with a small minority using incorrect boundaries or leaving gaps between the bars. Weaker responses showed all the classes the same width or used a scale which required the grid to be extended and this is not acceptable. The careful use of a ruler is essential to ensure that lines drawn are on the gridlines where necessary and along part squares where necessary. Graphs should always be drawn with a suitable pencil and a ruler for accurate positioning of lines. Axes should be carefully labelled, including appropriate units. Shading is not recommended as it often obscures the lines.

(b) Many candidates were able to write down the formula but did not apply it correctly. Better solutions listed the midpoints then showed their use in the calculation. Others incorrectly used either the class boundaries or the class width rather than midpoints. A number of candidates calculated the variance accurately but did not take the square root to find the standard deviation.

(c) Many candidates who gave an answer here provided the correct interval. Some gave 40–60 or 60–90 as the interval thinking that the upper quartile would be in the higher classes.

(d) Some candidates did not attempt this part. A significant number of who did assumed that, because the recorded times were 10 higher and 10 lower, they would balance each other out. A few candidates were not precise enough with their reasoning, saying that the new data items were in the same range without specifying the class intervals.

Question 4

(a) Many candidates gave reasoning that was difficult to follow, for example omitting steps in the working showing that $a = \frac{1}{5}$. Better responses contained a full method with all operations shown leading to *a*, *b* and *c*. Some candidates did not consider all of the possible outcomes and omitted the factor of 3 to account for the 3 non-biased coins landing on heads (or tails). Others tried to use the sum of probabilities as 1 giving them only one equation with two (or more) unknowns as there was not enough information in the table alone to find the value of a. There was some evidence that candidates who struggled to attempt this part left the rest of the question unanswered.

(b) Stronger candidates appreciated the need to show their calculation using their *b* and *c*.

(c) Clear solutions were often seen from those who attempted this part. The correct interpretation of ‘fewer than 3 occasions’ meant that candidates had to sum the probabilities of 0,1 and 2 occasions. Many candidates had one or two of these probabilities missing. In a very few cases the binomial coefficients were omitted. In a significant number of solutions the calculated binomial terms were rounded prematurely resulting in an incorrect final answer.

(d) This part was challenging for some candidates. Many used the correct probabilities of 0.8 and 0.2 but incorporated a binomial coefficient, while others used probabilities of $\frac{3}{80}$ and $\frac{77}{80}$. A proportion of candidates who successfully made use of the geometric distribution did not add their results or rounded their answer incorrectly.

Question 5

A minority of candidates were unable to make a start on this question. Solutions including a supporting diagram were more likely to be correct; this is to be encouraged.

(a) Correct solutions included the standardisation formula evaluated to at least 3 significant figures followed by accurate use of the normal distribution tables. Incorrect values of 1.5^2 and $\sqrt{1.5}$ were often seen, and some candidates wrongly used a continuity correction. Those who included a shaded diagram usually chose the correct probability to evaluate.

(b) Stronger responses used the given proportions to identify correctly the probabilities to use in their standardisation formulae. Using a clear diagram enabled them to form 2 equations in μ and σ, with the correct sign for the z value of 1.329. A number of candidates attempted to use the formula for finding standard deviation or tried to use the binomial distribution.

(c) The majority of candidates found this part challenging. The question required them to find the number of leaves further than 1 standard deviation from the mean at each end of the distribution. Many who started the question only found one tail or the number of leaves within 1 standard deviation of the mean. Candidates are reminded that they must show their working. This includes demonstrating that $z = \pm 1$ and calculations involving the standardisation formula. A significant number of candidates provided no evidence of standardisation to substantiate their answer and simply quoted a probability of 0.68 which was insufficient. Of those who multiplied their probability by 2000 to find the expected number of leaves, many used a decimal rounded to 3 significant figures (or fewer). Candidates should appreciate the need to use the full value obtained from the normal distribution table. The best solutions provided the answer as an integer with no reference to approximation.

Question 6

(a) The method most often adopted was to add the probability of completing first time to the probability of failing the first attempt and completing the second. In successful solutions a tree diagram was often seen. Where the answer is given, as in this question, there is a requirement to use mathematical operations clearly to produce the answer. Some candidates multiplied together all the probabilities.

(b) Some candidates attempted this question efficiently using the answer to **part (a)** and then multiplying it by the probability of completing level 2. The method most often seen was to find the probabilities of all 4 routes to complete the game. Most successful solutions made effective use of a tree diagram but some of these omitted one or more of the routes. Some candidates only included two terms, not realising that they needed to include the level 1 completion.

(c) Most candidates who attempted this part realised that they had to find the probability of finishing the game with exactly one fail at either level 1 or level 2. In many cases candidates were able to use some of their working from **part (b)**. Many incomplete solutions were seen where candidates neglected to find the conditional probability as requested. An exact fractional answer was rarely seen and some candidates who had a correct method rounded their final answer incorrectly.

MATHEMATICS

Paper 9709/52
Probability & Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively. When making an error, candidates would be well advised to cross out and replace the term as it is extremely difficult to interpret accurately terms that are overwritten.

Candidates should state only non-exact answers to 3 significant figures; exact answers should be stated exactly. It is important that candidates realise the need to work to at least 4 significant figures throughout to justify an answer to 3 significant figures. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. There is no requirement for probabilities to be stated as a decimal so candidates should leave their answer as the fraction obtained.

The interpretation of success criteria in binomial questions is an essential skill for this component, candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, in some responses it was difficult to follow the thinking within the solution as responses were not always set out clearly. The best solutions often included some simple notation to clarify the process that was being used.

Simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. The labelling of statistical diagrams has improved noticeably.

Candidates seem to have had sufficient time to complete all the work they were able to do, although some candidates may not have managed their time effectively. A few candidates did not appear to have prepared well for some topics, in particular applying the normal approximation in different contexts. Many clear and correct solutions were seen for **Questions 1** and **5**. The context in **Questions 4, 6** and **7** was found to be challenging for many candidates.

Comments on specific questions

Question 1

This coding question proved challenging for some candidates. Very few solutions showed an initial statement expanding the coding $\sum(x-200) \equiv \sum x - \sum 200$, which provides the basis for determining the value of *n* from the data provided. The majority of solutions simply formed an equation with the values provided, not always interpreting $\sum 200$ as $200n$. The most common error was to introduce *n* inappropriately, often linked with 446.

Question 2

Many good solutions to this standard probability question were seen. Candidates should be reminded that there is no expectation for probabilities to be stated as decimal values. Although not required by the question, an outcome space was often present in the most successful solutions.

(a) Most candidates correctly identified the possible outcomes. The most common error was the omission of 2. A few candidates used a standard dice which was incorrect and reduced the challenge of the question significantly. Candidates should be aware that the values in the probability distribution table must sum to 1 but many solutions did not satisfy this requirement.

(b) Most candidates used an appropriate process to calculate $E(X)$ and $Var(X)$. The best solutions stated the unsimplified expressions fully prior to evaluation, as required to support the accurate answers. Candidates who made errors in **part (a)** often did not provide sufficient evidence of their method in **part (b)**. Weaker solutions did not use an appropriate variance formula, either by omitting to square their $E(X)$ or squaring the probabilities rather than the outcomes.

Question 3

Some good solutions to this statistical data question were seen. Candidates are advised to use a ruler when constructing statistical diagrams accurately. A small number of candidates did not apply the key appropriately to the stem-and-leaf diagram provided, for example omitting the decimal point.

(a) Most candidates stated the median correctly. There was less consistency in the interquartile range, and significant variation in the quartiles found. In this component, the upper and lower quartiles are usually found as the value midway between the median and the maximum/minimum values.

(b) A surprisingly large number of candidates only constructed one box-and-whisker plot, which was often unidentified. The best solutions used a scale of 2 cm = 0.01 cm diameter, which allowed all the data values to be plotted accurately. More solutions than in the past had a fully labelled linear horizontal scale, including the units and 'diameter'. Candidates should be aware of the importance of communicating fully the information provided within the context. Many diagrams were drawn without a ruler, which limited the accuracy that could be achieved and is inappropriate at this level. The best diagrams identified companies *A* and *B* clearly, and ensure the lines were clearly visible.

(c) Comments in this component are expected to be related to the context, so simply comparing statistical values is not sufficient. With the data provided and the work undertaken in **part (b)**, comments relating to either the central tendency or spread were appropriate. The best comments often included a generalisation, such as 'in general, company *A* made pipes with a larger diameter' (central tendency) or 'company *B* produced pipes with a greater variety of diameters' (spread).

Question 4

This was found challenging by many and a significant proportion of candidates made no attempt. Good solutions often included a simple sketch of the normal curve to identify the required probability areas. A number of solutions lost accuracy because of premature approximation of *z*-values to 3 significant figures before finding the probability areas. Candidates should know to work to a higher degree of accuracy than required in the final answer, and to work to the greatest accuracy possible when using the normal tables.

(a) Many good solutions had a simple sketch to help interpret the criteria given for the required probability area. The best solutions recognised the alternative notation used to state the mean and variance of the distribution. A common error was to use $\sqrt{0.03}$ as the standard deviation rather than $\sqrt{0.03^2} = 0.03$ as expected. Better solutions showed the values substituted into the normal standardisation formula to determine the *z*-value then the tables used efficiently to find the two separate probability areas. $\Phi(0.333) - (1 - \Phi(1.333))$ was the most common method of determining the required value, although there was often confusion with signs. A number of candidates assumed that this was a symmetrical area, as in other recent papers, and did not evaluate the second standardisation formula but simply stated a *z*-value that was the negative of the first *z*-value.

(b) Candidates were required to use the information provided to find the probability of the given condition, and many found this challenging. They needed to equate the normal standardisation formula to an appropriate *z*-value and solve to find the unknown standard deviation. Again, successful solutions often included a sketch with the required probability area clearly identified. Many solutions were stated to 3 decimal places rather than 3 significant figures, an unexpected misunderstanding at this level.

Question 5

This question was successfully identified by most candidates as involving discrete random variables and so it linked to the binomial and geometric distributions. However, there was confusion seen in some responses as to the conditions required for the geometric distribution to be applied.

(a) Most candidates recognised that this as a binomial question although they needed to deduce the probability of success from information given in the question. Good solutions clearly stated all the unsimplified terms required and then evaluated without including intermediate steps. Where candidates gave intermediate values, they often lost accuracy through premature approximation. The most common error was to misinterpret the success criteria and calculate the probability that nine or more candidates will play at least one musical instrument. A less common, but still frequent, error was simply to calculate the probability that nine candidates will play at least one musical instrument.

The interpretation of success criteria is an important skill in this component, and candidates should be encouraged to focus on achieving a good understanding of the differences that occur.

(b) The best solutions included the test to show that the normal approximation was appropriate and gave unsimplified expressions for the mean and variance then evaluated them accurately without rounding. As the data was discrete, it was necessary to include a continuity correction when using the normal standardisation formula. It was also important to show values substituted into the formula. A simple sketch of the normal curve helped to identify the required probability area. Weaker responses either omitted the continuity correction or included 40 in the success criteria.

A number of solutions did not take account of the different success criteria but incorrectly used the same probabilities as **part (a)**. Other answers were incorrect because they were stated to 3 decimal places rather than the 3 significant figures required.

Question 6

Most candidates recognised that the question required the appropriate use of permutations and combinations.

(a) The majority of candidates successfully calculated the number of arrangements. A common error was to omit the division by 2!2! to account for the repeated Cs and Os.

(b) Most candidates subtracted the number of arrangements with Cs at the end and the Os together from the total number of arrangements with the Cs at the end. Candidates who assumed that the Cs were identifiable multiplied their expression by 2! incorrectly. The best solutions identified the scenario and then stated the unevaluated expression before calculating the difference. Weaker solutions simply stated the answer.

Some candidates used the alternative approach of considering the scenario C ^ ^ ^ ^ ^ C. These were more often successful in identifying the number of arrangements of the letters excluding the Os, then multiplying by the number of ways in which the Os could be inserted so they were not next to each other. Again, the most common error was to assume that the Cs were identifiable and therefore to multiply the expression by 2!. A less common, but frequent, error was to use ${}^{6}P_{2}$ rather than ${}^{6}C_{2}$ to determine the number of arrangements in which the Os could be inserted.

(c) Many candidates found this question challenging or omitted it. The most efficient approach was to identify the possible scenarios meeting the criteria and then calculate the possible number of outcomes for each scenario. The most common error was to assume that Cs and Os could be identified and therefore to multiply by the additional factor, with 80 the most common incorrect total. Many solutions omitted scenarios 2Cs 1O and 1C 2Os. A few solutions incorrectly included 0Cs 0Os in the possible scenarios.

A small number of candidates chose the alternative approach of subtracting the scenarios where the number of Cs and Os were equal from the total possible number of selections of 4. Many assumed that 2Cs and 2Os was the only possible scenario that fulfilled the criteria and did not consider 1C and 1O or 0Cs 0Os. Only the most confident candidates realised that it was also necessary to remove the repeated values for 1C 1O, 2Cs 1O and 1C 2Os.

(d) Candidates found this question to be the most demanding on the paper. To be answered successfully, they needed to consider the criteria given in the question and the effect of having the 2 Os. A few candidates did identify that the four possible conditions were: both Os in a group with a C; both Os in a group without a C; 1O in a group with a C and 1O in a group without a C; 1C 1O in two groups.

Most candidates simply extended the process started in **part (b)** but with the Cs in different groups. They calculated the number of ways of selecting the remaining letters as ${}^7C_2 \times {}^5C_2 \times {}^3C_3 = 210$, which was incorrect, then multiplied this by 3 as there were three ways to order C^^ C^^ ^^^.

Question 7

Most candidates identified that this was a probability question, with a few misinterpreting it as another permutation and combination question. The most common error was to assume that the eggs were replaced after being chosen, even though the question states that they were eaten so that the sweet could be identified. Candidates should be reminded that there is no expectation for probabilities to be stated as decimal values.

(a) Many good solutions included a tree diagram, which although complex often assisted in all parts of the question. Better responses identified the possible outcomes, showed the probability calculation and then added the three values. Weaker solutions assumed replacement and simply cubed the given probabilities. An unexpectedly high number of candidates used the appropriate numerators but did not reduce the denominator values.

(b) The most efficient approach was to use the values calculated in **part (a)** in the conditional probability formula. An error seen was to calculate P(3 Yellow sweets) × P(3 sweets the same) as the numerator, apparently a misunderstanding of conditional probability. It is unclear why a number of candidates used P(3 Orange sweets) as the numerator.

(c) A significant number of candidates did not attempt this question. The most efficient solutions recognised that P(at least 1 orange sweet) = 1 – P(0 orange sweets) using the techniques in **part (a)**. The most common error was to assume replacement. Solutions that were less structured often led to only some of the required scenarios being included in the calculations.

MATHEMATICS

Paper 9709/53
Probability & Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. This advice is particularly applicable to **Questions 6(b)** and **7(d)** where logical working helped candidates to reach the correct answer.

Candidates also need to make sure they read the question carefully, giving their answer in the required form. Many did not give the probabilities in **Question 3(a)** as numerical fractions. Others misinterpreted the changing conditions in different parts of **Question 7**.

Candidates are advised to consider whether an answer makes sense in the context of the question. This was particularly relevant to **Question 5(a)**.

General comments

Candidates often had difficulties in distinguishing between 3 significant figures and 3 decimal places in this paper. In **Questions 4(a)**, **4(c)**, and **6(c)** the first figure after the decimal point was a zero and, in all these cases, a number of candidates gave the answer to 3 decimal places which was only 2 significant figures.

If a final answer needs to be correct to 3 significant figures, candidates should work to at least 4 significant figures throughout to justify their final answer. This was the case in **Question 4(c)** where they were required to subtract the sum of three binomial terms from 1.

Comments on specific questions

Question 1

(a) Better responses took advantage of the size of the graph paper provided and chose an appropriate scale (2 cm to 25 minutes and 1 cm to 10 candidates) to make plotting easier. It was important to draw a curve (not a series of ruled lines) and to label the axes with ‘time’, ‘minutes’ and ‘cumulative frequency’. Those candidates who chose difficult scales were more likely to introduce errors.

(b) Candidates who produced fully correct answers found 20 per cent of 150 (30) then indicated their method by drawing a horizontal line from 30 candidates until it met the graph at about $t = 40$. Some incorrectly gave 30 as their final answer, while others read from the cumulative frequency of 20 or found 20 per cent of 160 rather than 150.

Question 2

(a) Most candidates accurately calculated the mean height by adding all the different values given and dividing by 10. A few candidates confused mean and median.

(b) Almost all the candidates knew that, with an even number of pieces of data, they needed to find the arithmetic mean of the 10^{th} and 11^{th} items (5.4 and 5.5 respectively).

(c) Correct responses referred to the presence of the extreme value 19.4 which would affect the mean not the median.

Question 3

(a) Candidates who reached correct probability values first calculated the probabilities in terms of k, equated the sum of all the probabilities to 1, solved to find $k = \frac{1}{18}$ then filled in the numerical probabilities. A significant number of candidates gave the probabilities in their table as multiples of k instead of as numerical fractions. Only a few candidates did not make use of $P(X = x) = kx^2$ and gave all their probabilities as $\frac{1}{4}$ or tried to change the x-values into terms involving k.

(b) This part of the question was answered well, with most candidates confidently calculating the mean and the variance correctly. Only a few candidates omitted to subtract the square of the mean from $E(X^2)$. Some candidates who gave their probabilities in terms of k in **part (a)** initially gave the answers to this part in terms of k again but could recover by obtaining numerical probabilities correctly and successfully using the formulae for $E(X)$ and $Var(X)$.

Question 4

(a) Most candidates recognised the geometric distribution and obtained the correct answer. However, some were confused about the difference between 3 significant figures and 3 decimal places in their answer. A common answer was 0.047 which is only 2 significant figures.

(b) A number of candidates correctly subtracted $\left(\frac{5}{6}\right)^5$ from 1. The most common error was to subtract $\left(\frac{5}{6}\right)^4$ instead because candidates misinterpreted 'no more than 5' as 'fewer than 5'. Many candidates chose the longer method of considering throws 1 to 5 separately: $\frac{1}{6}+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$ with some omitting the final term because they misinterpreted the wording of the question as above.

(c) Some candidates had difficulty in working out that there were three ways to obtain a total less than 4 with two dice. Most successful candidates listed the three ways (1,1), (1,2) and (2,1) and then found the required probability $\frac{3}{36} = \frac{1}{12}$. Some stopped at this point and presented $\frac{1}{12}$ as their final answer. Most recognised the need to use a binomial distribution with $n = 10$ and $p = \frac{1}{12}$ and successfully calculated 1 – P(0, 1 or 2). A few candidates rounded their binomial probabilities to 3 significant figures before subtracting the total from 1, resulting in an incorrect answer of 0.44.

Question 5

(a) The majority of candidates were able to demonstrate their knowledge of how to standardise the weights and use tables to find the correct z-values: –1.12 and 1.4. The more challenging part of the question was finding the correct area. Many candidates subtracted $\Phi(1.12)$ from $\Phi(1.4)$, obtaining an incorrect answer of 0.0506. In this question, it might have helped to question the likelihood of such a small probability, realising there was an error and that $\Phi(-1.12)$ is $1 - \Phi(1.12)$.

(b) Many candidates successfully calculated 20 000 × \$0.30 × (1 – $\Phi(1.4)$) and 20 000 × \$0.24 × their answer to **part (a)**, then added the two numbers. The most common errors were including apples weighing less than 142 grams, or using a price of \$0.24 for apples weighing less than 205 grams.

(c) Most candidates used the tables correctly to find a z-value of 0.583. Some did not appreciate that the z-value must be negative since the probability that the weight was more than w grams was 0.72 and so w must be smaller than the mean. A few candidates made an error in standardising by equating their formula to a probability rather than a z-value.

Question 6

(a) A large number of candidates drew the tree diagram correctly, but some only included two events instead of three and hence had incorrect probabilities.

(b) Better responses recognised that this was a conditional probability question which required them to divide P(one success) by P(at least one success). They obtained the correct final answer and set their work out clearly, showing three products added for the numerator: P(SFF) + P(FSF) + P(FFS), and the method used to find the denominator, P(at least one success). A number of candidates opted to find P(at least one success) by adding the seven 3-factor products that included at least one success instead of using 1 – P(FFF). This would have been more efficient and less prone to arithmetic errors. Some thought that P(one success) was the final answer. Candidates needed to state which probabilities they were finding, especially if their tree diagram is incorrect.

(c) Many candidates mistakenly assumed this was a binomial question because of the six jumps. Of those who realised it was not, some produced only a three-factor product ($0.2 \times 0.3 \times 0.3$) for the first three jumps as successes, but correctly worked out the probability of successes in the 4^{th} to 6^{th} jumps ($0.8 \times 0.9 \times 0.9 \times 0.1 \times 0.3 \times 0.3$). Few candidates realised that they needed to add the probabilities of the two outcomes. This was a question where confusion arose between 3 significant figures and 3 decimal places, frequently resulting in an answer of 0.016 instead of 0.0160.

Question 7

(a) Many correct responses were seen, using ${}^{12}C_5 \times {}^{7}C_4 \times {}^{3}C_3$ to find the correct answer. Alternative methods filling the vehicles in a different order are: ${}^{12}C_4 \times {}^{8}C_3 \times {}^{5}C_5$ or ${}^{12}C_3 \times {}^{9}C_4 \times {}^{5}C_5$. A common error was to add the combinations rather than multiply, and a few candidates wrongly used permutations instead of combinations. Another correct approach was to arrange the 12 people in 12! ways then divide by $5! \times 3! \times 4!$ as the order in which they sat in each vehicle did not matter.

(b) Most candidates appreciated the need to find a product of factorials when considering each family: 4! for the four remaining members of the Lizo family once Mr Lizo has gone first, 6! for the six members of the Kenny family, 2! for the Martin family and 2! for the Nantes family. The most common error was to forget that, if Mr Lizo goes first his family also goes first, hence only three families (Kenny, Martin and Nantes) can change order. So they needed to multiply by 3! (not 4!).

(c) Most realised that there were ${}^{7}C_4$ ways of selecting the four adults for the team. As only three families had children, the three chosen were the Martin child and one Kenny child (4 ways) and one Lizo child (3 ways), so $1 \times {}^{4}C_1 \times {}^{3}C_1 = 12$ ways of choosing the children. The majority of candidates multiplied ${}^{7}C_4$ by 12 but some made the same mistake as in **part (a)** and added them instead.

(d) Candidates who explained their work clearly were more likely to reach the right answer. The best responses used an elegant method which was to subtract the number of ways with neither Mr Kenny nor Mr Lizo from the total number of ways.

The words ‘at least one of Mr Kenny or Mr Lizo’ were confusing for some candidates who thought there could be more than one Mr Kenny or Mr Lizo. Others misread the question and thought that the condition that the children came from different families applied to **part (d)** as well as **part (c)**. Candidates need to be aware that, when a condition is within part of a question, it only applies to that part.

Of those who made good progress, the most common mistake was not to distinguish between ‘the number of ways with Mr Kenny’ and ‘the number of ways with Mr Kenny and not Mr Lizo’, or between ‘the number of ways with Mr Lizo’ and ‘the number of ways with Mr Lizo and not Mr Kenny’. If they chose to add the number of ways with Mr Kenny to the number of ways with Mr Lizo, they double counted the number of ways with Mr Kenny and Mr Lizo and so they needed to subtract the number of ways with both. If they chose to add the number of ways with Mr Kenny but not Mr Lizo and the number of ways with Mr Lizo but not Mr Kenny, they needed to add the number of ways with both. The most common error was to add the number of ways with both, the number of ways with Mr Kenny and the number of ways with Mr Lizo, so counting them both three times.

MATHEMATICS

Paper 9709/61
Probability & Statistics 2

Key messages

It is important for candidates to read the questions carefully.

Drawing a sketch or diagram can often help candidates to understand and apply the information given in the question.

For an answer to be fully correct, it needs to be supported by clear working, as stated in the rubric on the question paper.

In a hypothesis test, the conclusion should be written in the context of the question. It should use appropriate language with a degree of uncertainty, and it should not contain any contradiction.

General comments

Some candidates did not seem fully prepared for the demands of this paper. Candidates performed well on some questions (**Questions 1(a)**, **5(a)** and **6(a)**) but found other questions particularly demanding (**Questions 4(b)** and **6(d)**). There were a few places on the paper where it appeared that candidates had not read the question carefully. In **Question 6** some candidates appeared to show a lack of understanding with many candidates preferred to use standard methods rather than interpreting the scenario given. In such situations, candidates would be advised to draw a diagram which may help with interpretation.

Timing did not appear to be a problem, and presentation of work was generally acceptable.

Some comments on individual questions follow; these highlight common errors, but it should be noted that there were some exemplary solutions too.

Comments on specific questions

Question 1

(a) This question was relatively well attempted. Many candidates successfully found correct values for the unbiased estimate of the population mean and variance. Only a few candidates found the biased estimate of the population variance. The confidence interval was often successfully found though errors were seen including the use of an incorrect z value.

(b) Some candidates correctly stated the answer of 38 by finding 95 per cent of 40. Others incorrectly found 5 per cent or 97.5 per cent of 40 and some did not know what was required.

Question 2

Many candidates found this part to be challenging. When setting up the hypotheses, p should have been used; many candidates used μ or x. Few candidates found the correct binomial expression, and few made a valid comparison. Many candidates just calculated $P(X = 5)$ or used incorrect values of p and q in their binomial expression. Others attempted to use a normal distribution or a Poisson distribution. After clearly showing a valid comparison, the conclusion should have been written in context and with a level of uncertainty in the language used.

Question 3

Some candidates successfully found the required probability, but errors were often made, particularly when finding the variance. Standardising was generally well attempted, but not all candidates found the correct probability region. A common error was giving an answer greater than 0.5 rather than less than 0.5 and this could have been avoided by drawing a diagram.

Question 4

(a) Many candidates realised that the need to find the mean and variance of both X and Y from the distributions given. Many of them then found $E(X - 3Y)$ successfully, but made errors when finding $Var(X - 3Y)$, for example using 3 rather than 3^2 or subtracting rather than adding in the formula. The question asked for the standard deviation; many candidates left their final answer as the variance. Candidates need to read the question carefully.

(b) Many candidates found this part to be particularly challenging, with many candidates not realising that they needed to consider $X = 0$, $Y = 0$ and $X = 1$, $Y = 15$. Many tried incorrectly to work with a normal distribution and others were unable to make a start on the question.

Question 5

(a) This part was well attempted, with many candidates successfully finding the correct value for λ and using a correct expression (which should be fully shown) to find the required probability. Errors included using an incorrect value for λ or omitting a term in their Poisson expression.

(b) This part was not quite as well attempted, though many candidates used N(162, 162) and many attempted to standardise. Some candidates omitted to use a continuity correction or used incorrect ones. Attempting to find the required probability (the area between two z values) was not straightforward for some.

(c) The correct value for λ was 4.65, but some candidates did not find this value and instead used 2.4 (the value just for the trucks). Candidates are advised to read the question carefully. In this case, the probability of cars and trucks arriving in the 10-minute period was required. Those who successfully found λ usually went on to find the correct Poisson expression. Candidates with an incorrect value for λ were able to gain some credit for use of a relevant Poisson expression. It is important that this expression is fully shown.

Question 6

(a) Some candidates realised that the median was $\frac{a}{2}$, but a large number tried to find the median by integration. Some were successful but many did not use a correct expression for $f(x)$.

(b) Some candidates realised that the required probability was $\frac{1}{4}$, but again many tried to find this probability by integration. A diagram may have helped candidates here.

(c) In this part candidates needed to realise that $f(x)$ was $\frac{1}{a}$. Of those candidates who had this correct expression, many successfully found the variance. As this was a 'show that' question it was important that all working was shown, and the answer was correctly reached with no errors or omissions. Many candidates had an incorrect expression for $f(x)$ but could still apply a correct method.

(d) Very few candidates were successful in this part, and a significant number made no attempt at all. It would have been helpful for candidates to draw a diagram here and those that did were often successful or partially successful reaching the answer. A few candidates found the answer by integration, though this was a particularly long method.

Question 7

(a) When stating hypotheses, it is important to be precise. Candidates should state 'population mean' (not just mean) or μ. Errors included use of 27 rather than 28.2 or a mixture of these, and use of x rather than μ.

(b) A good number of candidates made a successful attempt at finding the probability of a Type I error. Many candidates standardised correctly and found the correct probability, though occasionally the $\sqrt{40}$ was omitted when standardising.

(c) Some candidates realised that they needed to find the conclusion drawn from the information given, i.e., that H_0 would not be rejected. The question asked if it would be possible to make a Type I or a Type II error or both. Many candidates merely said a Type II error could be made but did not say that a Type I could not. It is important for candidates to read the question carefully and answer it fully.

MATHEMATICS

Paper 9709/62
Probability & Statistics 2

Key messages

For an answer to be fully correct, it needs to be supported by clear working, as stated in the rubric on the question paper. Final answers should be to 3 significant figure accuracy.

In a hypothesis test, the conclusion should be written in the context of the question. It should use appropriate language with a degree of uncertainty, and it should not contain any contradiction.

General comments

In general, candidates demonstrated their knowledge in the situations presented. Questions that were done well were **Question 3(a)** and **Question 5(a)** and **(b)**, whilst **Question 4(b)** and **(c)** and **Question 5(c)** were found to be more demanding. Most candidates showed the required amount of working, but there were instances where some candidates' solutions lacked essential working.

Timing did not appear to be a problem, and presentation of work was generally acceptable.

There were some very good responses, but equally there were responses that demonstrated a lack of understanding for the demands of the paper.

Some comments on individual questions follow; these highlight common errors, but it should be noted that there were some complete and fully correct solutions too.

Comments on specific questions

Question 1

(a) This question was answered well by most candidates. Many were able to form the required confidence interval but there were some errors seen, mainly from use of an incorrect z value. Other errors arose through mixing standard deviation and variance or using n rather than the square root of n in the formula. Most candidates gave their answer as an interval as required.

(b) Candidates were required to comment that the sample had not been chosen randomly. Some candidates did so succinctly while others explained that the first 50 throws would not be representative and gave an acceptable reason why this might be the case. Some candidates did not give a sufficiently full explanation.

Question 2

There were many good responses to this question. Hypotheses were not always given in sufficient detail, for example, candidates should state 'population mean' rather than just 'mean' or alternatively use μ. Standardising was generally done well, but it was important to show clearly the comparison between the z value found and 1.96 (or equivalent comparison using areas) then draw a conclusion. The conclusion should be in context and with a level of uncertainty in the language used. Some candidates did not show a clear comparison or made an invalid comparison between an area and a z value. Other candidates gave a conclusion that was too definite or that did not reflect the context of the question.

Question 3

(a) A large number of candidates realised that a Poisson approximation was appropriate and successfully used $P_o(3.6)$ to find the required probability. Errors included adding an extra incorrect term in the Poisson expression (possibly from misinterpreting 'fewer than 3'). Some candidates did not use an approximating distribution at all but found the probability using B(200, 0.018).

(b) Candidates did not always justify fully their approximating distribution, instead quoting the conditions in general rather than applying them to the context of the question. For example, saying 'n large' or even '$n > 50$' was not enough. The value of n should be quoted from the information given (i.e. $n = 200$ and therefore $n > 50$) to justify fully the approximation used in **part (a)**. A similar statement is required for the value of np (or for q).

Question 4

(a) Most candidates were able to give correct hypotheses for the test, though some were not precise enough (for example stating $H_0 = 4.6$ rather than H_0: $\mu = 4.6$). Use of either λ or μ, and 4.6 or 9.2, was acceptable.

(b) Many candidates found this part to be challenging. Some candidates did not know how to find a critical region and even those who did were not always able complete the solution. To justify that the critical region was $X \leqslant 3$, it was important to calculate both $P(X \leqslant 3)$ and $P(X \leqslant 4)$, with all relevant working shown, then to compare the value with 0.02. Many candidates correctly used Po(9.2). A variety of errors was seen. Candidates often either calculated individual probabilities, e.g.
($P(X = 3)$, $P(X = 4)$, or calculated only $P(X \leqslant 3)$), or omitted to show the comparison with 0.02. A large number of candidates gave the critical region as a probability, $P(X \leqslant 3)$, or as 0.0184, and some candidates did not show all relevant working.

(c) Not all candidates realised that carrying out the test meant making a comparison then writing a conclusion. As the critical region had been calculated, the easiest comparison was to say 5 was not in the critical region (i.e. $5 > 3$). An alternative method was to compare $P(X \leqslant 5)$ with 0.02. Many candidates only calculated $P(X = 5)$, but comparing $P(X = 5)$ with 0.02 was not valid.

(d) Many candidates realised that they needed to look at their conclusion in **part (c)** to check whether H_0 had been accepted or rejected. It was important to refer specifically to this context; quoting the definition of a Type I error was not sufficient.

(e) This part of the question was generally answered well. Most candidates used N(276, 276) and standardised, though many made an error in the continuity correction or omitted it.

Question 5

(a) As this was a 'show that' question, it was important that all relevant working was shown. For a fully correct solution, there should be no errors in the working (for example missing factors of $\frac{3}{16}$ or missing brackets).

(b) Candidates generally knew how to find Var(X) although some of them did not subtract the square of the mean.

(c) Many candidates found this part to be challenging. Candidates using ($0.5 - P(3 < X < 4)$) or ($P(2 < X < 3) - 0.5$) were generally more successful than candidates attempting to find the median.

Question 6

(a) Many correct attempts were seen. Errors included finding an incorrect value for the variance or calculating an incorrect area i.e. finding an area > 0.5 rather than < 0.5.

(b) Again, this was well attempted. Candidates usually found E(T) successfully but made some errors finding Var(T) or in calculating an incorrect area as in **part (a)**.

Question 7

This question was generally well attempted. Weaker responses used the Poisson distribution, but many candidates correctly used N(2.9, $\frac{2.9}{100}$) and standardised. Of these, some candidates put in a correct continuity correction while others did not. Another approach used N(290, 290), again standardising with or without a continuity correction. There was some confusion between methods, with candidates using $\sqrt{100}$ when not required or omitting it when it was required. For both methods, a fully correct solution could be achieved with the correct continuity correction or without one.

MATHEMATICS

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Probability & Statistics 2

Key messages

For an answer to be fully correct, it needs to be supported by clear working, as stated in the rubric on the question paper. This means that individual terms need to be stated, for example binomial terms in **Question 2(b)** and **Question 2(c)** and Poisson terms in **Question 5(b)**.

In a hypothesis test, the conclusion should be written in the context of the question. It should use appropriate language with a degree of uncertainty, and it should not contain any contradiction. These requirements applied in **Question 3(b)**.

General comments

The comments below indicate common errors and misconceptions, however, it should be noted that there were also some strong and complete answers presented that demonstrated clear understanding of the syllabus topics.

Comments on specific questions

Question 1

Many candidates standardised correctly using the standard error $\frac{480}{10}$ and choosing the correct area for the probability.

Question 2

(a) Some very clear explanations of a Type I error were seen. For a fully correct response, candidates needed to refer to more than 10 per cent of left-handed students. They also needed to state whether or not the null hypothesis was valid, mention the idea of a conclusion from the test and refer to the context of the question.

(b) Many candidates used 1 – (the probability that 4 or fewer students were left-handed), listing all five binomial terms as required. This showed their understanding of a Type I error. Weaker responses omitted some of these terms or incorrectly used a Poisson distribution or normal distribution.

(c) Many candidates found the probability that 4 or fewer students were left-handed and listed the five binomial terms using B(20, 0.3) as required. Weaker responses omitted some of these terms so could not reach the correct answer.

Question 3

(a) Various reasons for using a sample rather than the whole population were acceptable, for example, ‘batteries would be unusable after testing’. Comments about the population were also acceptable, e.g. ‘the population would be too large to test’ or ‘testing the population would be too expensive’ or too time-consuming.

(b) Many candidates carried out the steps in the hypothesis test correctly and efficiently, stating the hypotheses, standardising, making a comparison and stating their conclusion in context. Some candidates made errors in one or more of these steps.

(c) A large number of fully correct confidence intervals were seen. Candidates used the correct z value of 1.881 or 1.882, stated the calculation they were carrying out and gave their answer as an interval, as required.

Question 4

(a) This question was tackled well by many candidates. They successfully found the new distribution for $S + R + B$ to be N(515, 74) and chose the correct area for the probability.

(b) Some clear and accurate solutions were seen from candidates who dealt correctly with the requirement that $S < 1.4R$ by changing this to $S - 1.4R < 0$. They found the correct distribution for $S - 1.4R$ as N(20, 94). Other candidates were unsuccessful because they did not use this new distribution or they did not use 1.4^2 when calculating the variance. In this situation, where '0' was not at the mean point of the normal graph, a diagram is helpful for choosing the correct area. Some candidates misunderstood this question by including the box in their calculations.

Question 5

(a) Many correct responses were seen in which candidates used a new parameter of 6.6 and calculated the probability for six clients using the Poisson formula. A few candidates incorrectly found the sum of a series of terms.

(b) Most candidates correctly used the parameter 2.2 and found 1 – (the probability of 4 or fewer clients), stating all five Poisson terms as required. Other candidates could not reach the correct answer since they omitted some terms.

(c) As the Poisson distribution for the 1-hour period yielded a parameter (26.4) which was greater than 15, most candidates realised that the appropriate approximating distribution was N(26.4, 26.4). This required a continuity correction which many candidates applied appropriately, using 19.5 instead of 20. Some candidates used the wrong continuity correction or no correction at all.

Question 6

(a) To find the mean of the five values, candidates needed to find the sum of the values and divide by 5. Some candidates incorrectly changed this expression to $\frac{13a}{5}$.

(b) This question gave the value of the unbiased population variance as 4. Candidates were required to substitute the given values into one of the standard formulae for the unbiased population variance. Those candidates who did this correctly could continue with the algebra to set up the quadratic equation in a then solve to find the value of a. Some candidates tried to use an incorrect formula and so could not find the equation nor the value of a. Other candidates started correctly but then incorrectly changed $47 + a^2$ into $47a^2$.

Question 7

(a) (i) The question asked candidates to write down the value of $P(X \geqslant 0)$ so it was sufficient to write '1'. Some candidates attempted integration but this was not required.

(ii) The question asked candidates to write down the value of $P(W \geqslant 0)$ so it was sufficient for them to write $\frac{1}{2}$. Again, some candidates attempted integration but this was not required.

(iii) It was sufficient for candidates to write '$q = \frac{p}{2}$'. Some candidates incorrectly wrote '$p = \frac{q}{2}$'.

(b) Candidates were required to use the given information that E(X) = 3 and the property that the total area under the graph of f(x) was 1. Many candidates correctly integrated xf(x) with limits 0 and a, in order to use E(X) = 3 then obtained the expression $p \quad a^4 = 12$ or an equivalent form. Some candidates also integrated f(x) with limits 0 and a and obtained the expression $p \quad a^3 = \frac{3}{2}$ or an equivalent form. These two expressions needed to be combined in order to find the value of a. Some candidates took an alternative approach, integrating g(w) with limits –a and a. To reach a correct final answer using this approach, they needed to use the correct connection between p and q obtained in **part (a)(iii)**.